4.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. List the intercepts of the equation $y = x^2 - 9$. (pp. 169–170)

2. Solve the equation: $2x^2 + 7x - 4 = 0$. (pp.96–99 and 101–105)

3. To complete the square of $x^2 - 5x$, you would add the number __________. (p. 99)

4. To graph $y = (x - 4)^2$, you would shift the graph of $y = x^2$ to the __________ a distance of __________ units. (pp. 262–271)

Concepts and Vocabulary

5. The graph of a quadratic function is called a(n) __________.

6. The vertical line passing through the vertex of a parabola is called the __________.

7. The x-coordinate of the vertex of $f(x) = ax^2 + bx + c$, $a \neq 0$, is __________.

8. *True or False:* The graph of $f(x) = 2x^2 + 3x - 4$ opens up.

9. *True or False:* The x-coordinate of the vertex of $f(x) = -x^2 + 4x + 5$ is $f(2)$.

10. *True or False:* If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$, $a \neq 0$, will touch the x-axis at its vertex.

Exercises

In Problems 11–18, match each graph to one the following functions without using a graphing utility.

11. $f(x) = x^2 - 1$

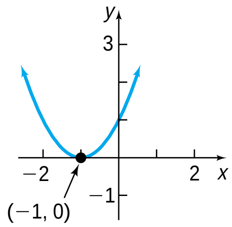
12. $f(x) = -x^2 - 1$

13. $f(x) = x^2 - 2x + 1$

14. $f(x) = x^2 + 2x + 1$

15. $f(x) = x^2 - 2x + 2$ **16.** $f(x) = x^2 + 2x$ **17.** $f(x) = x^2 - 2x$ **18.** $f(x) = x^2 + 2x + 2$

A.



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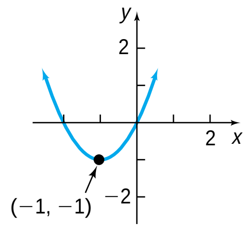

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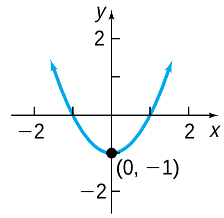

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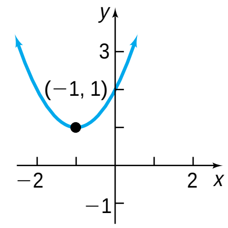

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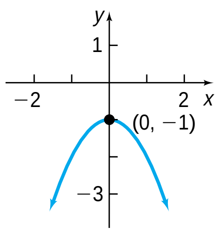

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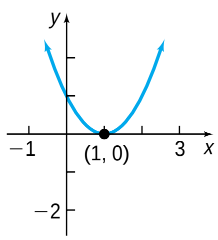

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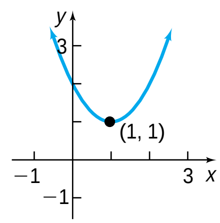

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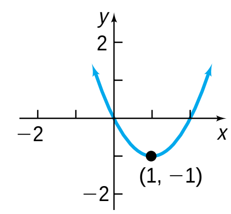


In Problems 19–34, graph the function f by starting with the graph of $y = x^2$ and using transformations (shifting, compressing, stretching, and/or reflection).

[**Hint:** If necessary, write f in the form $f(x) = a(x - h)^2 + k$.]

19. $f(x) = \frac{1}{4}x^2$ **20.** $f(x) = 2x^2$ **21.** $f(x) = \frac{1}{4}x^2 - 2$ **22.** $f(x) = 2x^2 - 3$

23. $f(x) = \frac{1}{4}x^2 + 2$ **24.** $f(x) = 2x^2 + 4$ **25.** $f(x) = \frac{1}{4}x^2 + 1$ **26.** $f(x) = -2x^2 - 2$

27. $f(x) = x^2 + 4x + 2$ **28.** $f(x) = x^2 - 6x - 1$ **29.** $f(x) = 2x^2 - 4x + 1$ **30.** $f(x) = 3x^2 + 6x$

31. $f(x) = -x^2 - 2x$ **32.** $f(x) = -2x^2 + 6x + 2$ **33.** $f(x) = \frac{1}{2}x^2 + x - 1$ **34.** $f(x) = \frac{2}{3}x^2 + \frac{4}{3}x - 1$

In Problems 35–52, graph each quadratic function by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, y-intercept, and x-intercepts, if any. Determine the domain and the range of the function. Determine where the function is increasing and where it is decreasing.

35. $f(x) = x^2 + 2x$ **36.** $f(x) = x^2 - 4x$ **37.** $f(x) = -x^2 - 6x$

38. $f(x) = -x^2 + 4x$ **39.** $f(x) = 2x^2 - 8x$ **40.** $f(x) = 3x^2 + 18x$

41. $f(x) = x^2 + 2x - 8$ **42.** $f(x) = x^2 - 2x - 3$ **43.** $f(x) = x^2 + 2x + 1$

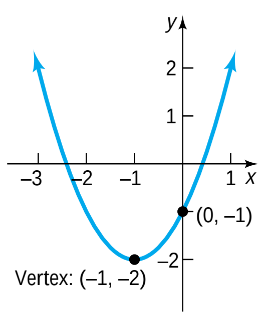
44. $f(x) = x^2 + 6x + 9$ **45.** $f(x) = 2x^2 - x + 2$ **46.** $f(x) = 4x^2 - 2x + 1$

47. $f(x) = -2x^2 + 2x - 3$ **48.** $f(x) = -3x^2 + 3x - 2$ **49.** $f(x) = 3x^2 + 6x + 2$

50. $f(x) = 2x^2 + 5x + 3$ **51.** $f(x) = -4x^2 - 6x + 2$ **52.** $f(x) = 3x^2 - 8x + 2$

In Problems 53–58, determine the quadratic function whose graph is given.

53.



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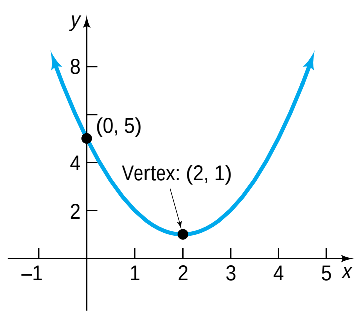

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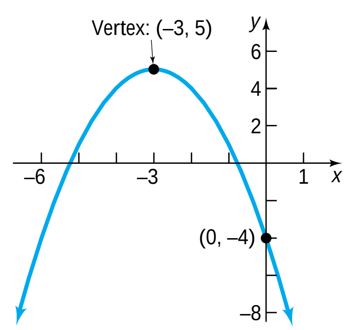

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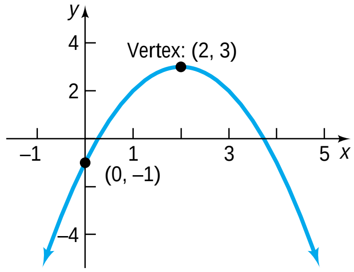

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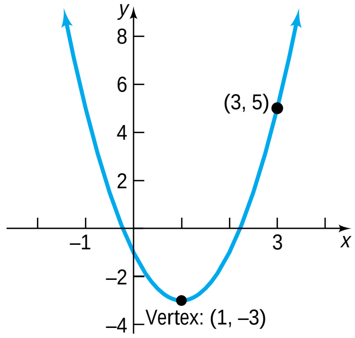

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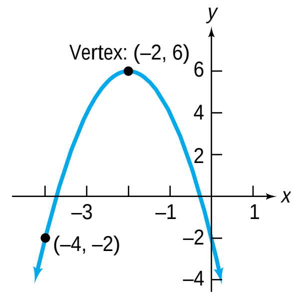


In Problems 59–66, determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find the value.

59. $f(x) = 2x^2 + 12x$ **60.** $f(x) = -2x^2 + 12x$ **61.** $f(x) = 2x^2 + 12x - 3$ **62.** $f(x) = 4x^2 - 8x + 3$

63. $f(x) = -x^2 + 10x - 4$ **64.** $f(x) = -2x^2 + 8x + 3$ **65.** $f(x) = -3x^2 + 12x + 1$ **66.** $f(x) = 4x^2 - 4x$

Answer Problems 67 and 68 using the following: A quadratic function of the form $f(x) = ax^2 + bx + c$ with $b^2 - 4ac > 0$ may also be written in the form $f(x) = a(x - r_1)(x - r_2)$, where r_1 and r_2 are the x-intercepts of the graph of the quadratic function.

67. (a) Find a quadratic function whose x-intercepts are -3 and 1 with $a = 1$; $a = 2$; $a = -2$; $a = 5$.
(b) How does the value of a affect the intercepts?
(c) How does the value of a affect the axis of symmetry?
(d) How does the value of a affect the vertex?
(e) Compare the x-coordinate of the vertex with the mid-point of the x-intercepts. What might you conclude?

68. (a) Find a quadratic function whose x-intercepts are -5 and 3 with $a = 1$; $a = 2$; $a = -2$; $a = 5$.
(b) How does the value of a affect the intercepts?
(c) How does the value of a affect the axis of symmetry?
(d) How does the value of a affect the vertex?
(e) Compare the x-coordinate of the vertex with the mid-point of the x-intercepts. What might you conclude?

69. Maximizing Revenue Suppose that the manufacturer of a gas clothes dryer has found that, when the unit price is p dollars, the revenue R (in dollars) is

$$R(p) = -4p^2 + 4000p$$

What unit price should be established for the dryer to maximize revenue? What is the maximum revenue?

70. Maximizing Revenue The John Deere company has found that the revenue from sales of heavy-duty tractors is a function of the unit price p that it charges. If the revenue R is

$$R(p) = -\frac{1}{2}p^2 + 1900p$$

what unit price p should be charged to maximize revenue? What is the maximum revenue?

71. Demand Equation The price p and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{6}x + 100, \qquad 0 \le x \le 600$$

(a) Express the revenue R as a function of x. (Remember, $R = xp$.)
(b) What is the revenue if 200 units are sold?
(c) What quantity x maximizes revenue? What is the maximum revenue?
(d) What price should the company charge to maximize revenue?

72. Demand Equation The price p and the quantity x sold of a certain product obey the demand equation

$$p = -\frac{1}{3}x + 100, \qquad 0 \le x \le 300$$

(a) Express the revenue R as a function of x.
(b) What is the revenue if 100 units are sold?
(c) What quantity x maximizes revenue? What is the maximum revenue?
(d) What price should the company charge to maximize revenue?

73. Demand Equation The price p and the quantity x sold of a certain product obey the demand equation

$$x = -5p + 100, \qquad 0 \le p \le 20$$

(a) Express the revenue R as a function of x.
(b) What is the revenue if 15 units are sold?
(c) What quantity x maximizes revenue? What is the maximum revenue?
(d) What price should the company charge to maximize revenue?

74. Demand Equation The price p and the quantity x sold of a certain product obey the demand equation

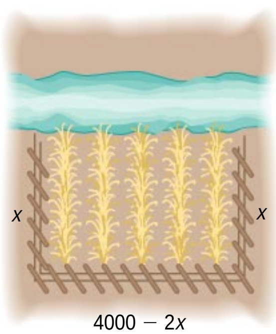
$$x = -20p + 500, \qquad 0 \le p \le 25$$

(a) Express the revenue R as a function of x.
(b) What is the revenue if 20 units are sold?
(c) What quantity x maximizes revenue? What is the maximum revenue?
(d) What price should the company charge to maximize revenue?

75. Enclosing a Rectangular Field David has available 400 yards of fencing and wishes to enclose a rectangular area.
(a) Express the area A of the rectangle as a function of the width w of the rectangle.
(b) For what value of w is the area largest?
(c) What is the maximum area?

76. Enclosing a Rectangular Field Beth has 3000 feet of fencing available to enclose a rectangular field.
(a) Express the area A of the rectangle as a function of x where x is the length of the rectangle.
(b) For what value of x is the area largest?
(c) What is the maximum area?

77. Enclosing the Most Area with a Fence A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed? (See the figure.)



78. Enclosing the Most Area with a Fence A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

79. Analyzing the Motion of a Projectile A projectile is fired from a cliff 200 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 50 feet per second. The height h of the projectile above the water is given by

$$h(x) = \frac{-32x^2}{(50)^2} + x + 200$$

where x is the horizontal distance of the projectile from the base of the cliff.
(a) How far from the base of the cliff is the height of the projectile a maximum?
(b) Find the maximum height of the projectile.
(c) How far from the base of the cliff will the projectile strike the water?
(d) Using a graphing utility, graph the function h, $0 \le x \le 200$.
(e) When the height of the projectile is 100 feet above the water, how far is it from the cliff?

80. Analyzing the Motion of a Projectile A projectile is fired at an inclination of 45° to the horizontal, with a muzzle velocity of 100 feet per second. The height h of the projectile is given by

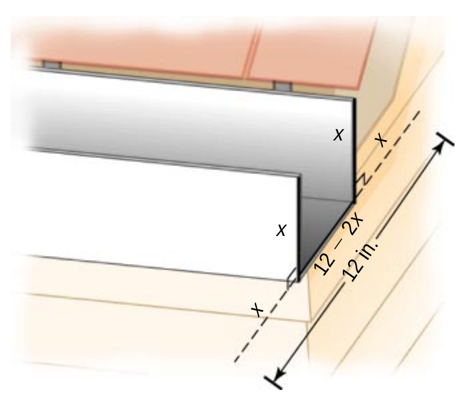
$$h(x) = \frac{-32x^2}{(100)^2} + x$$

where x is the horizontal distance of the projectile from the firing point.
(a) How far from the firing point is the height of the projectile a maximum?
(b) Find the maximum height of the projectile.
(c) How far from the firing point will the projectile strike the ground?
(d) Using a graphing utility, graph the function h, $0 \le x \le 350$.
(e) When the height of the projectile is 50 feet above the ground, how far has it traveled horizontally?

81. Suspension Bridge A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center. (Assume that the road is level.)

82. Architecture A parabolic arch has a span of 120 feet and a maximum height of 25 feet. Choose suitable rectangular coordinate axes and find the equation of the parabola. Then calculate the height of the arch at points 10 feet, 20 feet, and 40 feet from the center.

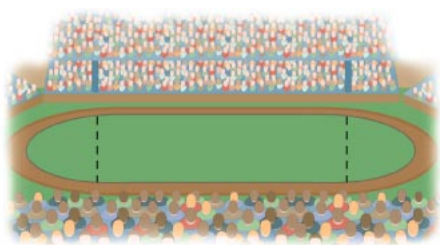
83. Constructing Rain Gutters A rain gutter is to be made of aluminum sheets that are 12 inches wide by turning up the edges 90°. What depth will provide maximum cross-sectional area and hence allow the most water to flow?



84. Norman Windows A Norman window has the shape of a rectangle surmounted by a semicircle of diameter equal to the width of the rectangle (see the figure). If the perimeter of the window is 20 feet, what dimensions will admit the most light (maximize the area)?

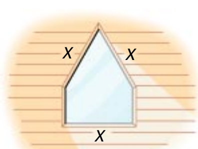
[**Hint:** Circumference of a circle $= 2\pi r$; area of a circle $= \pi r^2$, where r is the radius of the circle.]

85. Constructing a Stadium A track and field playing area is in the shape of a rectangle with semicircles at each end (see the figure). The inside perimeter of the track is to be 1500 meters. What should the dimensions of the rectangle be so that the area of the rectangle is a maximum?



86. Architecture A special window has the shape of a rectangle surmounted by an equilateral triangle (see the figure). If the perimeter of the window is 16 feet, what dimensions will admit the most light?

[**Hint:** Area of an equilateral triangle $= \left(\frac{\sqrt{3}}{4}\right)x^2$, where x is the length of a side of the triangle.]



87. Hunting The function $H(x) = -1.01x^2 + 114.3x + 451.0$ models the number of individuals who engage in hunting activities whose annual income is x thousand dollars.
SOURCE: Based on data obtained from the National Sporting Goods Association.

(a) What is the income level for which there are the most hunters? Approximately how many hunters earn this amount?

(b) Using a graphing utility, graph $H = H(x)$. Are the number of hunters increasing or decreasing for individuals earning between \$20,000 and \$40,000?

88. Advanced Degrees The function $P(x) = -0.008x^2 + 0.868x - 11.884$ models the percentage of the U.S. population whose age is given by x that has earned an advanced degree (more than a bachelor's degree) in March 2000.
SOURCE: Based on data obtained from the U.S. Census Bureau.

(a) What is the age for which the highest percentage of Americans have earned an advanced degree? What is the highest percentage?

(b) Using a graphing utility, graph $P = P(x)$. Is the percentage of Americans that have earned an advanced degree increasing or decreasing for individuals between the ages of 40 and 50?

89. Male Murder Victims The function $M(x) = 0.76x^2 - 107.00x + 3854.18$ models the number of male murder victims who are x years of age $(20 \le x < 90)$.
SOURCE: Based on data obtained from the Federal Bureau of Investigation.

(a) Use the model to approximate the number of male murder victims who are $x = 23$ years of age.

(b) At what age is the number of male murder victims 1456?

(c) Using a graphing utility, graph $M = M(x)$.

(d) Based on the graph drawn in part (c), describe what happens to the number of male murder victims as age increases.

90. Health Care Expenditures The function $H(x) = 0.004x^2 - 0.197x + 5.406$ models the percentage of total income that an individual that is x years of age spends on health care.
SOURCE: Based on data obtained from the Bureau of Labor Statistics.

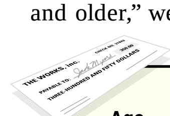
(a) Use the model to approximate the percentage of total income that an individual that is $x = 45$ years of age spends on health care.

(b) At what age is the percentage of income spent on health care 10%?

(c) Using a graphing utility, graph $H = H(x)$.

(d) Based on the graph drawn in part (c), describe what happens to the percentage of income spent on health care as individuals age.

91. Life Cycle Hypothesis An individual's income varies with his or her age. The following table shows the median income I of individuals of different age groups within the United States for 1995. For each age group, let the class midpoint represent the independent variable, x. For the class "65 years and older," we will assume that the class midpoint is 69.5.



Age	Class Midpoint, x	Median Income, I
15–24 years	19.5	\$20,979
25–34 years	29.5	\$34,701
35–44 years	39.5	\$43,465
45–54 years	49.5	\$48,058
55–64 years	59.5	\$38,077
65 years and older	69.5	\$19,096

SOURCE: U.S. Census Bureau

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) The quadratic function of best fit to these data is

$$I(x) = -42.6x^2 + 3806x - 38{,}526$$

Use this function to determine the age at which an individual can expect to earn the most income.
(c) Use the function to predict the peak income earned.
(d) Use a graphing utility to verify that the function given in part (b) is the quadratic function of best fit.
(e) With a graphing utility, draw a scatter diagram of the data and then graph the quadratic function of best fit on the scatter diagram.

92. Life Cycle Hypothesis An individual's income varies with his or her age. The following table shows the median income I of individuals of different age groups within the United States for 1996. For each age group, the class midpoint represents the independent variable, x. For the age group "65 years and older," we will assume that the class midpoint is 69.5.

Age	Class Midpoint, x	Median Income, I
15–24 years	19.5	\$21,438
25–34 years	29.5	\$35,888
35–44 years	39.5	\$44,420
45–54 years	49.5	\$50,472
55–64 years	59.5	\$39,815
65 years and older	69.5	\$19,448

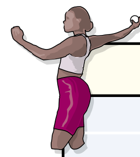
SOURCE: U.S. Census Bureau

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) The quadratic function of best fit to these data is

$$I(x) = -44.8x^2 + 4009x - 41392$$

Use this function to determine the age at which an individual can expect to earn the most income.
(c) Use the function to predict the peak income earned.
(d) Use a graphing utility to verify that the function given in part (b) is the quadratic function of best fit.
(e) With a graphing utility, draw a scatter diagram of the data and then graph the quadratic function of best fit on the scatter diagram.

93. Height of a Ball A shot-putter throws a ball at an inclination of 45° to the horizontal. The following data represent the height of the ball h at the instant that it has traveled x feet horizontally.



Distance, x	Height, h
20	25
40	40
60	55
80	65
100	71
120	77
140	77
160	75
180	71
200	64

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.
(b) The quadratic function of best fit to these data is

$$h(x) = -0.0037x^2 + 1.03x + 5.7$$

Use this function to determine how far the ball will travel before it reaches its maximum height.
(c) Use the function to find the maximum height of the ball.
(d) Use a graphing utility to verify that the function given in part (b) is the quadratic function of best fit.
(e) With a graphing utility, draw a scatter diagram of the data and then graph the quadratic function of best fit on the scatter diagram.

94. Miles per Gallon An engineer collects data showing the speed s of a Ford Taurus and its average miles per gallon, M. See the table.

Speed, s	Miles per Gallon, M
30	18
35	20
40	23
40	25
45	25
50	28
55	30
60	29
65	26
65	25
70	25

(a) Draw a scatter diagram of the data. Comment on the type of relation that may exist between the two variables.

(b) The quadratic function of best fit to these data is

$$M(s) = -0.018s^2 + 1.93s - 25.34$$

Use this function to determine the speed that maximizes miles per gallon.

(c) Use the function to predict miles per gallon for a speed of 63 miles per hour.

(d) Use a graphing utility to verify that the function given in part (b) is the quadratic function of best fit.

(e) With a graphing utility, draw a scatter diagram of the data and then graph the quadratic function of best fit on the scatter diagram.

95. Chemical Reactions A self-catalytic chemical reaction results in the formation of a compound that causes the formation ratio to increase. If the reaction rate V is given by

$$V(x) = kx(a - x), \qquad 0 \le x \le a$$

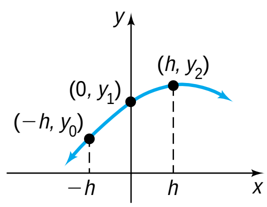
where k is a positive constant, a is the initial amount of the compound, and x is the variable amount of the compound, for what value of x is the reaction rate a maximum?

96. Calculus: Simpson's Rule The figure shows the graph of $y = ax^2 + bx + c$. Suppose that the points $(-h, y_0)$, $(0, y_1)$, and (h, y_2) are on the graph. It can be shown that the area enclosed by the parabola, the x-axis, and the lines $x = -h$ and $x = h$ is

$$\text{Area} = \frac{h}{3}(2ah^2 + 6c)$$

Show that this area may also be given by

$$\text{Area} = \frac{h}{3}(y_0 + 4y_1 + y_2)$$



97. Use the result obtained in Problem 96 to find the area enclosed by $f(x) = -5x^2 + 8$, the x-axis, and the lines $x = -1$ and $x = 1$.

98. Use the result obtained in Problem 96 to find the area enclosed by $f(x) = 2x^2 + 8$, the x-axis, and the lines $x = -2$ and $x = 2$.

99. Use the result obtained in Problem 96 to find the area enclosed by $f(x) = x^2 + 3x + 5$, the x-axis, and the lines $x = -4$ and $x = 4$.

100. Use the result obtained in Problem 96 to find the area enclosed by $f(x) = -x^2 + x + 4$, the x-axis, and the lines $x = -1$ and $x = 1$.

101. A rectangle has one vertex on the line $y = 10 - x$, $x > 0$, another at the origin, one on the positive x-axis, and one on the positive y-axis. Find the largest area A that can be enclosed by the rectangle.

102. Let $f(x) = ax^2 + bx + c$, where a, b, and c are odd integers. If x is an integer, show that $f(x)$ must be an odd integer.
[**Hint:** x is either an even integer or an odd integer.]

103. Make up a quadratic function that opens down and has only one x-intercept. Compare yours with others in the class. What are the similarities? What are the differences?

104. On one set of coordinate axes, graph the family of parabolas $f(x) = x^2 + 2x + c$ for $c = -3$, $c = 0$, and $c = 1$. Describe the characteristics of a member of this family.

105. On one set of coordinate axes, graph the family of parabolas $f(x) = x^2 + bx + 1$ for $b = -4$, $b = 0$, and $b = 4$. Describe the general characteristics of this family.

106. State the circumstances that cause the graph of a quadratic function $f(x) = ax^2 + bx + c$ to have no x-intercepts.

107. Why does the graph of a quadratic function open up if $a > 0$ and down if $a < 0$?

108. Refer to Example 8 on page 301. Notice that if the price charged for the calculators is \$0 or \$140 the revenue is \$0. It is easy to explain why revenue would be \$0 if the priced charged is \$0, but how can revenue be \$0 if the price charged is \$140?

'Are You Prepared?' Answers

1. $(0, -9), (-3, 0), (3, 0)$ **2.** $\left\{-4, \frac{1}{2}\right\}$

3. $\frac{25}{4}$ **4.** right; 4