The family of Gaussian Hypergeometric Distributions is characterized by its probability generating function, given by the Gaussian hypergeometric function (except for the constant)

$$ {}_2F_1\left(\alpha, \beta, \gamma, \lambda z\right) = \sum_{r=0}^{\infty} \frac{(\alpha)_r (\beta)_r}{(\gamma)_r} \frac{(\lambda z)^r}{r!}. \tag{1.1} $$

The probability mass function is

$$ f_r = P\left[X = r\right] = f_0 \frac{(\alpha)_r (\beta)_r}{(\gamma)_r} \frac{\lambda^r}{r!}, \qquad r = 0, 1, ..., \tag{1.2} $$

where f_0 is the constant of normalization. We denote these distributions as $GHD\left(\alpha, \beta, \gamma, \lambda\right)$.