As an example of this type of results, it can be proved that if  $\gamma > \beta$ , the  $GHDI(\alpha, \beta, \gamma, \lambda)$  is the mixture

$$Poisson(\Lambda) \bigwedge_{\Lambda} Gamma\left(\alpha, \frac{\lambda(1-P)}{1-\lambda(1-P)}\right) \bigwedge_{P} GBeta\left(\gamma - \alpha - \beta, \beta, \alpha, \lambda\right),$$
(2.4)

where  $GBeta(\gamma - \alpha - \beta, \beta, \alpha, \lambda)$  denotes a generalization of the Beta distribution whose density function is

$$f_P(p) = \frac{1}{{}_2F_1(\alpha,\beta;\gamma;\lambda)} \frac{\Gamma(\gamma)}{\Gamma(\gamma-\beta)\Gamma(\beta)} \frac{p^{\gamma-\beta-1}(1-p)^{\beta-1}}{(1-\lambda(1-p))^{\alpha}}, \quad 0 \le p \le 1.$$
(2.5)