

Scheme of Work

Cambridge Primary

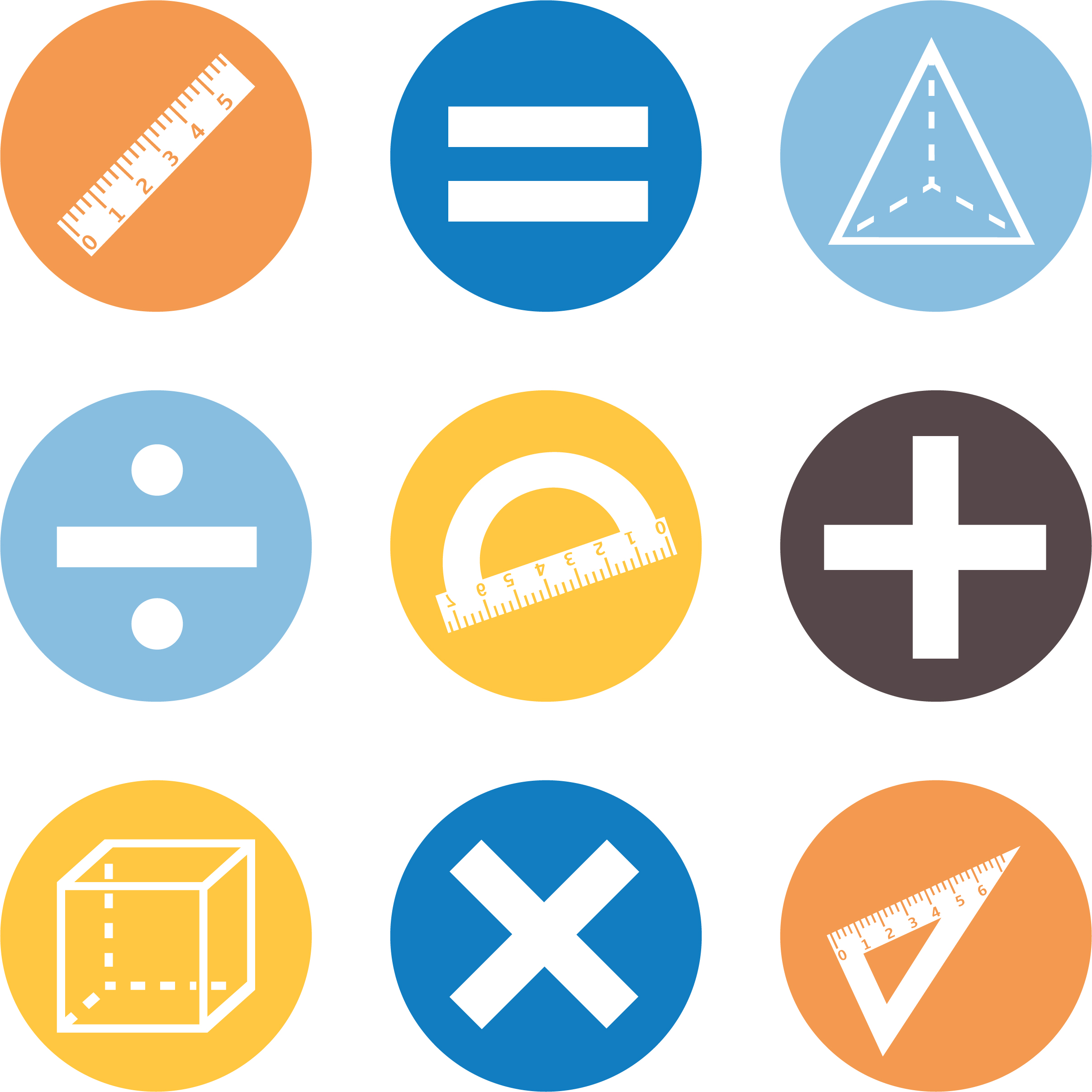
Mathematics 0096

Stage 3

This Cambridge Scheme of Work is for use with the Cambridge Primary

Mathematics Curriculum Framework published in September

2020 for first teaching in September 2021.



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**Changes to this Scheme of Work**

For information about changes to this Scheme of Work, go to page 86.

The latest Scheme of Work is version 2.0, published January 2021.

# Introduction

This document is a scheme of work created by Cambridge Assessment International Education for Cambridge Primary Mathematics Stage 3.

It contains:

* suggested units showing how the learning objectives in the curriculum framework can be grouped and ordered
* at least one suggested teaching activity for each learning objective
* a list of subject-specific language that will be useful for your learners
* common misconceptions
* sample lesson plans
* links to relevant NRICH activities to enrich learners’ mathematical experiences, **https://nrich.maths.org/**

You do not need to use the ideas in this scheme of work to teach Cambridge Primary Mathematics Stage 3. Instead use them as a starting point for your planning and adapt them to suit the requirements of your school and the needs of your learners. The schemes of work are designed to indicate the types of activities you might use, and the intended depth and breadth of each learning objective. These activities are not designed to fill all the teaching time for this stage. You should use other activities with a similar level of difficulty, for example, those from endorsed resources.

The accompanying teacher guide for Cambridge Primary Mathematics suggests effective teaching and learning approaches. You can use this scheme of work as a starting point for your planning, adapting it to suit the requirements of your school and needs of your learners.

## Long-term plan

This long-term plan shows the units in this scheme of work and a suggestion of how long to spend teaching each one. The suggested teaching time is based on learners having about 4 to 5 hours of Mathematics per week (about 120 to 150 hours per stage). The actual number of teaching hours may vary according to your context.

| Unit | Suggested teaching time |
| --- | --- |
| **Unit 3.1** Numbers to 1000, addition and subtraction | 17% (25 hours) |
| **Unit 3.2** Time | 10% (15 hours) |
| **Unit 3.3** Shapes and angles | 13% (20 hours) |
| **Unit 3.4** Patterns, place value and rounding | 10% (15 hours) |
| **Unit 3.5** Measurement | 10% (15 hours) |
| **Unit 3.6** Multiplication and division | 10% (15 hours) |
| **Unit 3.7** Fractions | 17% (25 hours) |
| **Unit 3.8** Statistical methods and chance | 13% (20 hours) |
| **Total** | **150 hours** |

## Sample lesson plans

You will find two sample lesson plans at the end of this scheme of work. They are designed to illustrate how the suggested activities in this document can be turned into lessons. They are written in more detail than you would use for your own lesson plans. The Cambridge Primary Mathematics Teacher Guide has information on creating lesson plans.

## Other support for teaching Cambridge Primary Mathematics Stage 3

Cambridge Primary centres receive access to a range of resources when they register. The Cambridge Primary support site at [**https://primary.cambridgeinternational.org**](https://primary.cambridgeinternational.org) is a password-protected website that is the source of the majority of Cambridge-produced resources for the programme. Ask the Cambridge Coordinator or Exams Officer in your school if you do not already have a log-in for this support site.

Included on this support site are:

* the Cambridge Primary Mathematics Curriculum Framework, which contains the learning objectives that provide a structure for your teaching and learning
* grids showing the progression of learning objectives across stages
* the Cambridge Primary Mathematics Teacher Guide, which will help you to implement Cambridge Primary Mathematics in your school
* templates for planning
* worksheets for short teacher training activities that link to the teacher guide
* assessments provided by Cambridge
* a list of endorsed resources, which have been through a detailed quality assurance process to make sure they are suitable for schools teaching Cambridge Primary Mathematics worldwide
* links to online communities of Cambridge Primary teachers.

## Resources for the activities in this scheme of work

We have assumed that you will have access to these resources:

* paper, pens and pencils for learners to use
* rulers, set squares, protractors and calculators.

Other suggested resources for individual units and/or activities are described in the rest of this document. You can swap these for other resources that are available in your school.

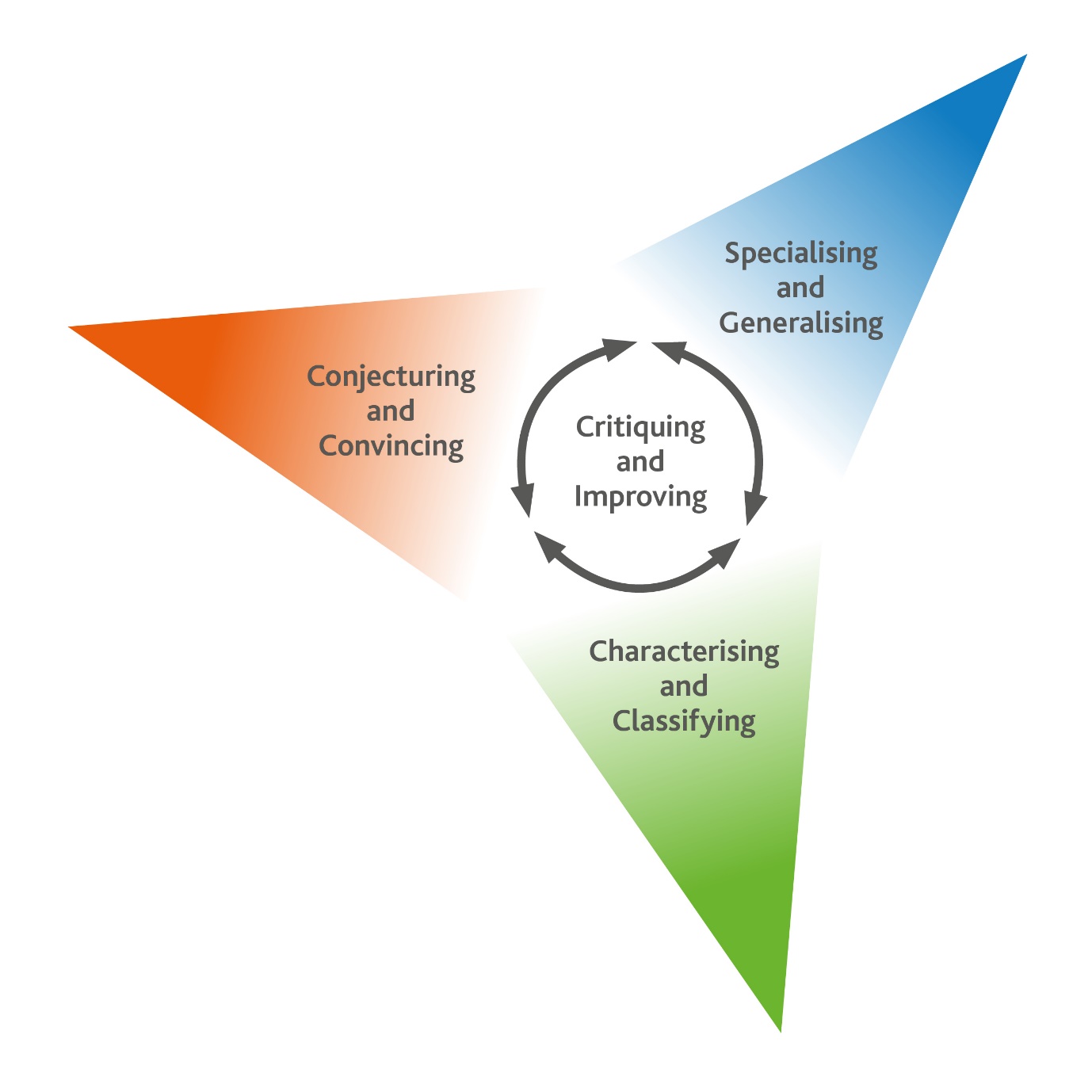
## Websites

We recommend NRICH to support Cambridge Primary Mathematics at [**https://nrich.maths.org/**](https://nrich.maths.org/)

NRICH publishes free and challenging mathematics activities for learners of all ages. The resources assist teachers to embed thinking and working mathematically with mathematics content. NRICH is based in both the University of Cambridge's Faculty of Education and the Centre for Mathematical Sciences.

There are many excellent online resources suitable for teaching Cambridge Primary Mathematics. Since these are updated frequently, and many are only available in some countries, we recommend that you and your colleagues identify and share resources that you have found to be effective for your learners.

## Approaches to teaching Cambridge Primary Mathematics Stage 3



Thinking and Working Mathematically

Thinking and Working Mathematically supports the mathematical concepts and skills in all strands of the Cambridge Primary Mathematics curriculum. When learners think and work mathematically, they actively engage with their learning of mathematics. They try to make sense of ideas and build connections between different facts, procedures and concepts. Learners who do not think and work mathematically can carry out processes that their teacher has shown them, but they may not understand why the processes work or what the results mean. Noticing inconsistencies, patterns and particular representations encourages learners to think and work mathematically. Practice, reflection and questioning will help them to improve.

Thinking and Working Mathematically has eight characteristics that are presented in four pairs:

* Specialising and Generalising
* Conjecturing and Convincing
* Characterising and Classifying
* Critiquing and Improving.

The eight Thinking and Working Mathematically characteristics are all closely connected and interdependent. A high-quality mathematics task may include one or more of them. The characteristics provide learners with the language they need to think and work mathematically. Learners can then decide what mathematical knowledge, procedures and strategies to use in order to gain a deeper understanding of mathematical questions.

Throughout this scheme of work, there are examples of classroom activities that link the Thinking and Working Mathematically characteristics with content learning objectives. We recommend you use the ideas in these examples to create further classroom activities.

| Thinking and Working Mathematically characteristics: | | Unit 3.1 | Unit 3.2 | Unit 3.3 | Unit 3.4 | Unit 3.5 | Unit 3.6 | Unit 3.7 | Unit 3.8 |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **TWM.01** | **Specialising** – Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | ✓ |  | ✓ |  |  | ✓ |  | ✓ |
| **TWM.02** | **Generalising** – Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria | ✓ | ✓ |  | ✓ |  | ✓ |  |  |
| **TWM.03** | **Conjecturing** – Forming mathematical questions or ideas |  |  |  | ✓ | ✓ |  | ✓ | ✓ |
| **TWM.04** | **Convincing** – Presenting evidence to *justify or challenge* a mathematical idea or solution | ✓ | ✓ |  | ✓ | ✓ | ✓ | ✓ |  |
| **TWM.05** | **Characterising** – Identifying and describing the mathematical properties of an object |  |  | ✓ | ✓ | ✓ |  |  |  |
| **TWM.06** | **Classifying** – Organising objects into groups according to their mathematical properties |  |  | ✓ |  |  |  |  | ✓ |
| **TWM.07** | **Critiquing** – Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | ✓ |  |  |  | ✓ | ✓ | ✓ |  |
| **TWM.08** | **Improving** – Refining mathematical ideas or representations to develop a more effective approach or solution | ✓ |  |  |  | ✓ |  | ✓ |  |

Misconceptions

Mathematical misconceptions are usually incorrect generalisations made by learners. Misconceptions should not be avoided, but instead used for teaching purposes to reveal learners’ thinking. Research suggests that asking learners open-ended questions about mathematical concepts is the most appropriate way to uncover misconceptions. Once a learner’s misconceptions have been identified, the next step is to know how to correct them. One approach is to give learners a variety of mathematical strategies to draw upon when finding solutions so that they can gain a deeper understanding of each mathematical concept.

Mental strategies and calculators

Mental calculation is a skill needed for everyday life, especially when paper or calculators are not available. Mental calculation relies on working memory, the organisation of thoughts and the use of efficient mathematical strategies when solving mathematical computations. It is important for learners to practise mental calculations and have a range of strategies as this improves understanding and recall as well as increasing confidence and proficiency.

Calculators are useful teaching aides. Although learners need to practise doing mental and written calculations, calculators can help them to notice patterns. They are also useful when learners are solving problems where non-calculator calculations would take the focus away from strategies. When well used, calculators can help learners to learn about numbers and the number system. Use calculators as a teaching aid to promote mental calculation and mental strategies and to explore mathematical patterns. Learners should understand when it is best to use calculators to help them calculate, and when to calculate mentally or using written methods.

As Cambridge International includes calculator-based assessments at Stages 5, 6, 7, 8 and 9, we recommend that learners begin to use calculators for performing and checking calculations from Stage 4. At Stages 5 and 6, learners should be developing effective use of calculators so that they are familiar with the buttons and functions of a basic calculator.

# Unit 3.1 Numbers to 1000, addition and subtraction

| Learning objectives covered in Unit 3.1 and topic summary: | | 3.1 Topic 1  Understanding numbers | 3.1 Topic 2  Using addition and subtraction facts | 3.1 Topic 3  Strategies for addition and subtraction | Thinking and Working Mathematically |
| --- | --- | --- | --- | --- | --- |
| **3Nc.01** | Estimate the number of objects or people (up to 1000). | ✓ |  |  | **TWM.04 Convincing** |
| **3Nc.03** | Use knowledge of even and odd numbers up to 10 to recognise and sort numbers. | ✓ |  |  | **TWM.02 Generalising** |
| **3Nc.04** | Recognise the use of an object to represent an unknown quantity in addition and subtraction calculations. |  | ✓ |  | **TWM.01 Specialising** |
| **3Nm.01** | Interpret money notation for currencies that use a decimal point. | ✓ |  |  |  |
| **3Nm.02** | Add and subtract amounts of money to give change. |  |  | ✓ |  |
| **3Ni.01** | Recite, read and write number names and whole numbers (from 0 to 1000). | ✓ |  |  | **TWM.04 Convincing** |
| **3Ni.02** | Understand the commutative and associative properties of addition, and use these to simplify calculations. |  | ✓ |  | **TWM.07 Critiquing**  **TWM.08 Improving** |
| **3Ni.03** | Recognise complements of 100 and complements of multiples of 10 or 100 (up to 1000). |  | ✓ |  | **TWM.07 Critiquing** |
| **3Ni.04** | Estimate, add and subtract whole numbers with up to three digits (regrouping of ones or tens). |  |  | ✓ | **TWM.04 Convincing** |
| **3Np.04** | Understand the relative size of quantities to compare and order 3-digit positive numbers, using the symbols =, > and <. | ✓ |  |  |  |

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| Unit 3.1 Topic 1 Understanding numbers |
| Outline of topic: |
| Learners will recite, read and write number names and whole numbers and develop their understanding of the relative size of quantities by comparing and ordering 3-digit numbers. They will improve their skills of estimating objects, people and numbers.  Learners will develop their understanding of odd and even numbers and they will begin to interpret decimal notation in currency. |
| Language: |
| **Key vocabulary:**  place value, digit  odd, even  money, coins, notes, currency  compare, order, smaller than, greater than, equal to  estimate, approximate  **Key phrases:**  Estimate the value of …  Guess how many … |
| Recommended prior knowledge: |
| * Estimation * Odd and even numbers * Understanding of money * Reciting, reading and writing numbers to 100 * Understand the relationship between addition and subtraction * Order and compare 2-digit numbers |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nc.01** Estimate the number of objects or people (up to 1000).  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Show learners a small container filled with 10 - 50 bottle tops. Ask learners to estimate how many bottle tops are in the container and discuss what strategies they used to estimate.  *What other examples can you think of where you might need to estimate a similar quantity?*  Ask learners to think of an example where they might estimate a number up to about 100. They might offer examples such as the leaves on a branch, the number of words in a paragraph etc.  Discuss the strategies that learners used to make their estimates and ask them to justify why they think their estimates were reasonable.  Learners will show they are **convincing (TWM.04)** when they can justify why their estimate is reasonable.  Show learners a larger container filled with 100 or more bottle tops. Discuss the differences between this container and the smaller container. Ask learners:   * *How much bigger, how much wider is it?* * *How many bottle tops might be in this container?*   Discuss their strategies for estimating the number of bottle tops such as counting a small section of bottle tops in the container and scaling up. Then ask learners to give further examples where the quantity is approximately 100.  Now ask learners to estimate a number up to 1000, e.g. learners on the playground or in their school.  Explain that one strategy for making an estimate is counting a smaller quantity and then scaling up using multiplication or repeated addition.  Select learners who offer unreasonable estimates and ask them to justify their answer. Ask other learners to offer suggestions on how to improve the estimates.  Finally ask learners:  *Where might you find:*   * *50 people?* * *100 objects?* * *500 - 800 people?* * *Less than 20 objects?* * *Approximately 1000 people or objects?*   **Resources:**  Examples of quantities to estimate up to 100 and up to 1000 such as bottle tops in containers. | **Possible misconceptions:**  Learners often try to calculate the answer immediately rather than offer an estimate first. |
| **3Nc.03** Use knowledge of even and odd numbers up to 10 to recognise and sort numbers.  **TWM.02 Generalising**  Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria | Remind learners that an even number of objects can be shared into 2 equal groups whereas an odd number of objects cannot.  In pairs, give learners a set of 0-9 digit cards each. Taking turns, each learner turns over one card at a time and decides if it is odd or even. Learners place the numbers into two piles: odd and even. Ask learners:  *Which pile will you put zero in?*  Give learners a selection of cards with odd and even numbers between 0 and 1000 and ask them to place them in the Carroll Diagram below:   |  |  |  | | --- | --- | --- | |  | Odd numbers | Not odd numbers | | Even numbers | - | 0, 256, 348 | | Not even numbers | 123, 571 | - |   *What do you notice about the numbers in the Carroll Diagram?*  *Can you explain why?*  Learners will show they are **generalising (TWM.02)** when they make general statements about odd and even numbers. For example, each number can only be odd or even it can never be both.  Ask learners to write the characteristics of odd and even numbers from 0 -1000. Even numbers always have the digit 0, 2, 4, 6 or 8 in the ones place. Odd numbers always have the digit 1, 3, 5, 7 or 9 in the ones place.  **Resources:**  0-9 digit cards  Cards with odd and even numbers between 0 and 1000 | **Possible misconceptions:**  Learners sometimes think that zero is not a number. Discuss with learners that while zero sometimes has no value, this is a quantity in itself. |
| **3Nm.01** Interpret money notation for currencies that use a decimal point. | Explain to learners that some countries use decimal notation when writing quantities of money.  Ask questions such as:   * *How many cents is $6.05?* * *Write 435 cents in dollars ($) and cents (c).* * *Write in dollars and cents the total of ten dollar coins and five 5c coins.*   Give learners the following amounts to choose from:  $2.45, $2.81, 513 cents, 357 cents, 325 cents, 383 cents, $3.69, $3.46, $3.05  Ask learners to choose which amounts match these statements:   * *I am less than five dollars.* * *I am not the same value as 346 cents.* * *I have a dollar sign.* * *If you changed me to cents, there would be an odd number of cents.* * *My value is less than $3.80.* * *My value is more than 350 cents.* * *My value is more than half of five dollars.* * *My value is less than 500 cents.* * *My value is three dollars and five cents.*   *How do you know if your answers are correct?*  Learners should explain how they decided which amounts matched each statement. | Wherever possible try to use real currency when teaching money.  You can initially use your own currency before introducing dollars and cents. For these examples, there are 100 cents in 1 dollar.  **Possible misconceptions:**  Learners sometimes confuse the place of the zero so write 3 hundred and 5 as 3005 or 35 instead of 305. When working with money, learners might not realise the zero in amounts like 3 dollars 5 cents and so write it as $3.5 or $3.50 rather than $3.05. |
| **3Ni.01** Recite, read and write number names and whole numbers (from 0 to 1000).  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners to fill in the missing spaces in the table below:   |  |  | | --- | --- | | **Number names** | **Numerals** | | Three hundred and four |  | |  | 650 | | Eight hundred and twenty-two |  | |  | 789 |   Ask learners to create their own tables and swap with a partner.  Using digit cards 0-9, select three cards. Ask learners to find all three-digit numbers that can be made from the cards.  *Write the largest, smallest, and one of the middle numbers in words.*  *How do you know if you have found all the three-digit numbers?*  Learners will show they are **convincing (TWM.04)** when they can justify why they think they have found all the possible solutions.  **Resources:**  0-9 digit cards | **Possible misconceptions:**  Learners may think that 3 tens + 2 hundreds + 8 ones is 328 not 238.  Learners sometimes confuse the place of the zero so write 3 hundred and 4 as 3004 or 34 instead of 304.  Learners sometimes do not remember the correct place value representation of individual digits, for example, they might write three hundred and fifty-six as 30056. |
| **3Np.04** Understand the relative size of quantities to compare and order 3-digit positive numbers, using the symbols =, > and <. | Ask learners questions such as:   * *Which is greater: 716 or 761?* * *Which is longer: 359m or 395m?* * *Yuri has walked 486 metres and Lily has walked 468 metres. Who has walked the furthest?*   Ask learners to use the symbols =, > and < to represent these questions accurately. For example, 716 < 761 and 761 > 716.  Ask learners to create their own questions and swap them with a partner to record mathematically.  *If 360 < ☐ < 390, what numbers could ☐ be?*  Ask learners to use the symbols to compare the heights of these famous buildings:   * Big Ben 96m * Shanghai Tower 632m * Eiffel Tower 300m * Empire State Building 381m * Burj Khalifa 828m   Ask learners to research the heights of mountains and put them in order of size using the =, > and < symbols.  **Resources:**  Access to computers, encyclopaedias or atlases to research heights of mountains. | **Possible misconceptions:**  Learners sometimes do not think about the place value of each digit.  Learners sometimes confuse the place of the zero so they write 3 hundred and 4 as 3004 or 34 instead of 304.  Learners sometimes do not remember the place value representation of individual digits, for example, they might write 3 hundred and fifty-six as 300506.  Learners sometimes confuse the two symbols < and >. Reinforce strategies for remembering them, such as > is like an arrow that points to the smaller number. |

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| Unit 3.1 Topic 2 Using addition and subtraction facts |
| Outline of topic: |
| Learners will recognise complements to 100 and complements of multiples of 10 or 100.  Learners will develop their understanding of the laws of arithmetic and understand that number sentences can be written using known and unknown quantities. |
| Language: |
| **Key vocabulary:**  place value, digit  number bonds, complements, pairs of numbers, multiples  commutative, associate  known, unknown  **Key phrases:**  What is the unknown quantity in this number sentence?  What is the known quantity in this number sentence?  Which number do we need to make this complement to 100? |
| Recommended prior knowledge: |
| * Complements to 10 * Understand the relationship between addition and subtraction |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nc.04** Recognise the use of an object to represent an unknown quantity in addition and subtraction calculations.  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | Remind learners that the = sign means equivalence by relating it to a balance. Use balance scales to model the number sentence by putting 4 and 7 counters on one side and 11 on the other side. The total of the sum can be on the left or right of the = sign, for example:  4 + 7 = ☐ or ☐ = 4 + 7  7 + ☐ = 11 or 11 = 7 + ☐  Write this number sentence on the board and ask learners to solve it:  7 + 5 = ☐ + 4  How many learners answer with 12? 8?  Discuss what the equals sign means and that each side of the balance scale must be equal. To emphasise the concept of balance ask them to also write the number sentence as:  ☐ + 4 = 7 + 5  Use balance scales to model the number sentence by putting seven and five counters on one side, four on the other side. What must be added to the four counters to make the scales balance?  Draw a bar model to show this representation a different way:   |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |   7 + 5 = 12  ☐ + 4 = 12  Ask learners to create their own number sentences where at least one part is missing and then swap with a partner.  Ask learners to write both addition and subtraction number sentences where the missing number is in different positions.  Learners will show they are **specialising (TWM.01)** by testing examples to see if they satisfy the mathematical equation.  **Resources:**  Balance scales  Counters | An unknown quantity is a quantity that can vary.  **Possible misconceptions:**  Learners sometimes find abstraction a difficult concept to understand. So, you can represent the equation with balance scales, if available, and the unknown quantity as an empty envelope, thus helping to provide a concrete representation for the abstract concept.  The balance is a useful analogy to link to with the representation of number sentences. Learners sometimes incorrectly believe that the final answer always comes directly after the = sign.  Learners sometimes incorrectly believe that the missing number is always after the = sign, so it is important to move the position of the missing number box. |
| **3Ni.02** Understand the commutative and associative properties of addition, and use these to simplify calculations.  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages  **TWM.08 Improving**  Refining mathematical ideas or representations to develop a more effective approach or solution | Ask learners to select three 0-9 digit cards and add the total.  For example, Samira turns over 3, 6 and 7 and the total is 16.  *How many ways can Samira add these numbers?*  Elicit from learners that Samira can add the numbers in the order that they were selected or in another order that she prefers.  For example:  3 + 6 + 7 = 16  6 + 7 + 3 = 16  7 + 3 + 6 = 16  These examples show that numbers in any addition question can be added together in any order and still result in the same total. This is called the commutative law. This law does not apply to subtraction or division.  Samira also notices that 3 and 7 are complements to 10 and so she uses this fact to make the addition question easier and simpler to solve.  + 6 = 10 + 6 = 16 or 6 + = 16 or 16 = + 6  This example demonstrates the associative law that learners can use when adding numbers together.  Learners will show they are **critiquing (TWM.07)** by comparing and evaluating different approaches to calculations, based on their arithmetic knowledge.  Using 0-9 digit cards ask learners to turn over three numbers and use the commutative and associate laws to simplify calculations. Ask them to write their answers showing which law they have used. For extra challenge, ask learners to generate three two-digit numbers and again use the commutative and associative laws of addition to make the questions easier to solve.  Learners will show they are **improving (TWM.08)** when they consider how they can change the order of the numbers in the number sentences to refine and simplify the calculations.  **Resources**  0-9 digit cards | Encourage learners to use mental calculations to add and subtract numbers where possible.  At this stage, learners have not been introduced to brackets so introduce associative property of addition informally  e.g. if 2 + 3 + 5 = 10 then  2 + 3 + 5 = + 5 = 5 + 5 = 10 or  2 + = 2 + 8 = 10  **Possible misconceptions:**  Learners may apply the laws incorrectly and think they apply to all operations. For example, wrongly assuming 45 – 5 = 5 – 45.  Addition and multiplication follow the commutative and associate laws but subtraction and division do not (except in a few special cases). |
| **3Ni.03** Recognise complements of 100 and complements of multiples of 10 or 100 (up to 1000).  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | In pairs give learners a pack of 0-9 digit cards and ask them to select two cards to combine to make a number and then find the complement to 100. For example, they select 1 and 2 so their number is 12 and the complement to 100 is 88.  Ask them to record their strategy.  For example, did they use a number line and count up to 100?  Did they work out the number to the next multiple of 10 and then count in 10s to 100?  Did they just know the complement?  Learners will show they are **critiquing (TWM.07)** when they compare and evaluate different approaches to calculations, based on their arithmetic knowledge.  After they have calculated five complements to 100, ask learners to share their strategies with the class. Record several on the board and discuss which are efficient, unusual or creative?  Now ask learners to choose one card from the digits 0-9 and then multiply the digit by 100 to generate a multiple of 100. Ask learners to find the complement to 1000.  **Resources:**  0-9 digit cards | Encourage learners to use mental calculations to find complements where possible.  **Possible misconceptions:**  Learners sometimes do not understand the relationship between multiples of 1, 10 and 100, so they might know that 3 + 7 = 10 but they are not able to transfer this knowledge to know that 30 + 70 = 100. |

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| Unit 3.1 Topic 3 Strategies for addition and subtraction |
| Outline of topic: |
| Learners will add and subtract amounts of money and will explore estimating before adding and subtracting whole numbers. |
| Language: |
| **Key vocabulary:**  add, subtract, equals, number sentence  money, coins, value, change  estimate  **Key phrases:**  Estimate the value of …  How many is …?  How do you know?  How much change do you have? |
| Recommended prior knowledge: |
| * Addition and subtraction of 2-digit numbers * Make simple estimates of numbers |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nm.02** Add and subtract amounts of money to give change. | Give learners a selection of problems to solve, such as the following:  *If it costs $1.50 for a child to swim, how much does it cost for two children? How much change will they receive from $5?*  *Mia has a 50 cent coin and three 20 cent coins. She pays 90 cents for a bag of sweets. How much money does she now have?*  *Mia bought three packets of sweets at 80 cents each. What change will she receive from $3?*  *You have $5. Some toys are priced at $2.90, $1.10, $2.20 and $1.50. Investigate how many toys you could buy with $5. For each purchase, how much change will you have?*  Ask learners to record their answers.  **Resources:**  Selection of word problems involving currency | Make sure learners are reading and saying money correctly, e.g. $1.50 is one dollar and fifty cents and not one point five or one and five.  For these examples, you can use your own currency before introducing dollars and cents.  **Possible misconceptions:**  Learners can sometimes struggle to work out change involving both dollars and cents (or the equivalent local currency). For example, if the change from $5 is $1.50 learners can sometimes ignore the decimal point and record it as $15 or 15 cents. |
| **3Ni.04** Estimate, add and subtract whole numbers with up to three digits (regrouping of ones or tens).  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners which number sentences below are true or false. Ask them to estimate first before calculating the answer.  386 + 12 = 402  987 – 225 = 762  405 – 60 = 345  672 + 145 = 817  Ask them to justify their estimates and answers, e.g. for the first example, my estimate is 386 + 10 = 396 so the answer is going to be false.  Learners will show they are **convincing (TWM.04)** when they can explain the strategies they used to estimate and why the statements are true or false.  Ask learners to make up their own true and false number sentences, swap with a partner and solve. | Understanding how to compose, decompose and regroup numbers in many different ways is essential for learners to become effective at calculating.  For both addition and subtraction calculations, use examples that do not require regrouping before introducing examples that do require regrouping.  For subtraction questions, initially do not use numbers with zeros e.g. 207 – 29.  **Possible misconceptions:**  Learners decompose numbers but forget to add (or regroup) all the different parts. |

# Unit 3.2 Time

| Learning objectives covered in Unit 3.2 and topic summary: | | 3.2 Topic 1  Reading and recording time | 3.2 Topic 2  Time problems | Thinking and Working Mathematically |
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| **3Gt.01** | Choose the appropriate unit of time for familiar activities. |  | ✓ | **TWM.04 Convincing** |
| **3Gt.02** | Read and record time accurately in digital notation (12-hour) and on analogue clocks. | ✓ |  | **TWM.04 Convincing**  **TWM.02 Generalising** |
| **3Gt.03** | Interpret and use the information in timetables (12-hour clock). | ✓ |  |  |
| **3Gt.04** | Understand the difference between a time and a time interval. Find time intervals between the same units in days, weeks, months and years. |  | ✓ |  |

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| Unit 3.2 Topic 1 Reading and recording time |
| Outline of topic: |
| Learners will read and record time in digital format and on analogue clocks. They will interpret information in timetables using the 12-hour clock. |
| Language: |
| **Key vocabulary:**  time, clocks, analogue, digital, minutes, hours, a.m., p.m.  timetables, information, data  **Key phrases:**  What time is …?  How long …? |
| Recommended prior knowledge: |
| * Read and record the time to five minutes in digital and analogue clock * Interpret and use the information in calendars |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
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| **3Gt.02** Read and record time accurately in digital notation (12-hour) and on analogue clocks.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Explain that the minute hand from an analogue clock has fallen off.  Analogue clock with hour hand pointing to 10  Ask learners:  *Can you estimate the time using the position of the hour hand only as it moves between two numbers?*  For example, if the hour hand is pointing half-way between the 10 and the 11, we can estimate that the time is about half past ten.  Ask learners to find the times on the examples below.  Analogue clock with hour hand pointing to 7Analogue clock with hour hand pointing to 5Analogue clock with hour hand pointing to 1  Give learners further images of analogue clock faces with missing minute hands and ask them to draw the minute hand in the appropriate position, then, underneath write the estimated time.  Learners will show they are **convincing (TWM.04)** when they are able to use language to convince a partner what the time on the clocks could be. For example, the hour hand has not yet reached the half-way point, so it is before half-past six.  **Resources:**  Images of clocks with the minute hand missing | **Possible misconceptions:**  Learners sometimes forget which hands the hour hand and the minute hand are. The double scale of a clock can confuse learners because the 1, for example, can represent 1 and 5.  Ensure learners understand the minute hand counts round in 5 but the hour hand counts round in 1s. |
| **3Gt.02** Read and record time accurately in digital notation (12-hour) and on analogue clocks.  **TWM.02 Generalising**  Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria | Explain that an animal needs to be given medicine every 50 minutes throughout the day. The time interval between each dose of medicine is 50 minutes. The first dose is given to the animal at 7:00a.m.  Show learners the first five times of the day when the animal needs to be given medicine:  7:00a.m., 7:50a.m., 8:40a.m., 9:30a.m., 10:20a.m.  Read the times aloud and ask the class to say the times too.  Ask learners:   * *Do you notice a pattern in the times?* * *Can you use this pattern to work out when the next dose will be given?*   Learners will show they are **generalising (TWM.02)** when they notice a pattern in the times of day and use this to find the following times when the animal needs to be given medicine. | **Possible misconceptions:**  Some countries use the notation a.m. (morning) and p.m. (afternoon or evening) when using a 12-hour clock. Learners sometimes forget the difference between a.m. and p.m. |
| **3Gt.03** Interpret and use the information in timetables (12-hour clock). | This timetable shows the times that buses arrive at Bus Stop A.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | 8:00a.m. | 10:30a.m. | 1:00p.m. | 3:30p.m. | 5:00p.m. |  * *Explain that Jamilla arrives at Bus Stop A at 2:50p.m. How long does she have to wait for a bus?* * *The 10:30a.m. bus arrives 35 minutes late. What time does it arrive?* * *How long is it between the first and last bus?*   Ask learners:  *What other questions can you ask someone about the times of the buses?*  Here is another timetable. A bus takes 20 minutes between each stop. Ask learners to complete the timetable.   |  |  |  |  | | --- | --- | --- | --- | | Main Street | 11:05 |  | 1:45 | | Football Field |  |  |  | | Shop |  | 1:05 |  | | School |  |  |  | | **Possible misconceptions:**  Some countries use the notation a.m. (morning) and p.m. (afternoon or evening) when using a 12-hour clock. Learners sometimes forget the difference between a.m. and p.m. |

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| Unit 3.2 Topic 2 Time problems |
| Outline of topic: |
| Learners will choose appropriate units of time for activities and be able to understand the difference between a time and a time interval. |
| Language: |
| **Key vocabulary:**  time, clocks, analogue, digital, minutes, hours, a.m., p.m.  time, time interval  **Key phrases:**  What is the time interval for …?  Which unit of time would you use here? |
| Recommended prior knowledge: |
| * Order and compare units of time (seconds, minutes, hours, days, weeks, months, years) |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Gt.01** Choose the appropriate unit of time for familiar activities.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners to suggest activities that they would estimate in the following units of time: hours, minutes, seconds, days, weeks, months, years.  Compare their examples.  Ask learners:  *Suggest a unit of time to estimate and then calculate:*   * *The time it takes you to eat lunch.* * *The time you watch television each week.* * *How long it is until the end of the year.*   Ask learners:   * *How long do you think it would take to boil water in a small pan?*   *3 minutes or 30 minutes?*   * *How long would it take you to walk across the road?*   *1, 10 or 100 seconds?*  Learners will be **convincing (TWM.04)** when they can justify why it would take 3 minutes or 30 minutes to boil water.  Ask learners to suggest their own activities and then ask the class how long they think the activity would take to complete. | **Possible misconceptions:**  Learners may use base ten instead of base 60 when thinking about time. |
| **3Gt.04** Understand the difference between a time and a time interval. Find time intervals between the same units in days, weeks, months and years. | Remind learners that a time interval is the difference between two events measured in time, or how long an event takes to complete. The interval can be in minutes, hours, days, weeks, months and years.  Ask learners to think of an activity that would take:   * One minute * One hour * One day * One week * One month * One year   A time interval is not the same as a time. For example:  *If a football match starts at 10a.m. and finishes at 10.45a.m., how long did the match last?* (Answer: 45 minutes is the time interval)  Travel is a good example to show the difference between a time and a time interval.  For example, the bus might leave at 10.30a.m. but the bus takes 30 minutes to get into the town, showing a time interval of 30 minutes.  Give learners some problems to solve:  *Amhed arrived at the shopping mall at 11am. He spent 45 minutes shopping.*  *How long did Amhed spend at the mall and what time did he leave?*  *Safia’s favourite television programme starts at 3.30. She arrives home at 2.30. How long will she have to wait until the programme starts?* (Answer: 1 hour is the time interval)  Ask learners to think of other time word problems for another learner to solve. | **Possible misconceptions:**  Learners sometimes forget that a time interval is different to reading a time. |

# Unit 3.3 Shapes and angles

| Learning objectives covered in Unit 3.3 and topic summary: | | 3.3 Topic 1  2D and 3D shapes | 3.3 Topic 2  Symmetry and reflection | 3.3 Topic 3  Angles, direction and position | Thinking and Working Mathematically |
| --- | --- | --- | --- | --- | --- |
| **3Gg.01** | Identify, describe, classify, name and sketch 2D shapes by their properties. Differentiate between regular and irregular polygons. | ✓ |  |  | **TWM.06 Classifying**  **TWM.01 Specialising** |
| **3Gg.05** | Identify, describe, sort, name and sketch 3D shapes by their properties. | ✓ |  |  | **TWM.05 Characterising** |
| **3Gg.08** | Recognise pictures, drawings and diagrams of 3D shapes. | ✓ |  |  | **TWM.05 Characterising** |
| **3Gg.09** | Identify both horizontal and vertical lines of symmetry on 2D shapes and patterns. |  | ✓ |  | **TWM.01 Specialising** |
| **3Gg.10** | Compare angles with a right angle. Recognise that a straight line is equivalent to two right angles or a half turn. |  |  | ✓ | **TWM.05 Characterising** |
| **3Gp.01** | Interpret and create descriptions of position, direction and movement, including reference to cardinal points. |  |  | ✓ | **TWM.05 Characterising** |
| **3Gp.02** | Sketch the reflection of a 2D shape in a horizontal or vertical mirror line, including where the mirror line is the edge of the shape. |  | ✓ |  |  |

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| Unit 3.3 Topic 1 2D and 3D shapes |
| Outline of topic: |
| Learners will develop their understanding of 2D and 3D shapes. They will identify, classify, name, sort and sketch regular and irregular polygons and be able to recognise 3D shapes in pictures, drawings and diagrams. |
| Language: |
| **Key vocabulary:**  polygons, 2D, 3D  square, rectangle, triangle, quadrilateral, pentagon, hexagon  cube, cuboid, prism, pyramid  vertex, vertices, face, edge  sketch, draw, sort, identify, describe, name  **Key phrases:**  What are the properties of this shape?  A … has … edges and … faces. |
| Recommended prior knowledge: |
| * Be able to identify 2D and 3D shapes from their properties * Recognise 2D and 3D shapes in different positions and orientations |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Gg.01** Identify, describe, classify, name and sketch 2D shapes by their properties. Differentiate between regular and irregular polygons.  **TWM.06 Classifying**  Organising objects into groups according to their mathematical properties  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | Give each learner a set of polygons, for example:  Ask learners to place the polygons in the correct box in the table below:   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | Number of edges | | | | |  | 3 | 4 | 5 | 6 | | Regular polygon |  |  |  |  | | Irregular polygon |  |  |  |  |   Learners will show they are **classifying (TWM.06)** when they identify the properties of each polygon to correctly place it in the table.  Now ask learners to draw a polygon in the three empty boxes in the table.  Learners will show they are **specialising (TWM.01)** when they choose an example of a polygon and check to see whether it satisfies the criteria given in the table.  This activity can be extended by asking learners:  *In which boxes can you put more than one possible shape? Why?*  **Resources:**  Polygon shapes | **Possible misconceptions:**  Learners may confuse the vocabulary of shape. The term ‘regular’ means specifically that each edge is the same length and each angle is the same. So, for example, a regular rectangle is a square. Learners often confuse regular with the everyday word meaning ‘normal’ or ‘usual’. |
| **3Gg.05** Identify, describe, sort, name and sketch 3D shapes by their properties.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Give learners drinking straws and sticky tape and ask them to make shapes for a cube, cuboid, pyramid and prism or ask them to sketch the shapes.  Once they have made their 3D models ask learners to complete the following table.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Shape** | **Cube** | **Cuboid** | **Pyramid** | **Prism** | | Vertices |  |  |  |  | | Faces |  |  |  |  | | Edges |  |  |  |  |   Learners will show they are **characterising (TWM.05)** when they identify the properties of each 3D shape.  Put a 3D shape into a bag without learners knowing which one it is, and invite them to ask questions to guess what it is. However, you can only answer yes or no.  For example: *Does the shape have equal faces?*  Put a shape into a bag and ask learners to feel it, describe the shape by feel to other learners making specific reference to its properties. Other learners then try to guess what the shape is.  **Resources:**  Drinking straws  Sticky tape  3D shapes | **Possible misconceptions:**  Learners may incorrectly refer to the faces of a shape as sides.  Learners may confuse prisms with pyramids. It is important that learners understand that prisms have the same cross-section all the way through, while pyramids do not as the faces come to a point (vertex). Ask learners: *What is the same and different about a triangular-based pyramid and a triangular prism?*  Learners sometimes incorrectly believe that cones are pyramids because of the point (vertex), but they are not. |
| **3Gg.08** Recognise pictures, drawings and diagrams of 3D shapes.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Show learners pictures of famous buildings.  For example:   * The Pyramids in Egypt * Oriental Pearl Tower in China * Big Ben in London   Ask them to identify the 3D shapes within these buildings.  Learners will show they are **characterising (TWM.05)** when they identify several properties such as faces, vertices or edges to describe various 3D shapes in buildings.  Ask learners to draw their own building, using as many 3D shapes as possible.  Take learners on a shape walk around the school. Ask them to identify and sketch 3D shapes on their walk. | **Possible misconceptions:**  Learners may refer to curved surfaces as faces when describing 3D objects. Faces of objects are always flat. Cylinders have curved surfaces and cubes have faces. |

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| Unit 3.3 Topic 2 Symmetry and reflection |
| Outline of topic: |
| Learners will develop their understanding of line (mirror) symmetry by identifying lines of symmetry in shapes and patterns and also by sketching reflections of 2D shapes using mirrors. |
| Language: |
| **Key vocabulary:**  symmetry, line symmetry, horizontal, vertical  mirror line, reflection  **Key phrases:**  How many lines of symmetry has a ….?  Can you see the reflection?  How does the mirror help you to see the reflection of a 2D shape? |
| Recommended prior knowledge: |
| * Identify lines of symmetry on 2D shapes * Recognise symmetrical patterns |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Gg.09** Identify both horizontal and vertical lines of symmetry on 2D shapes and patterns.  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | Show learners a set of polygons with horizontal and/or vertical lines of symmetry, for example:  Ask learners to predict which 2D shapes have horizontal and vertical lines of symmetry.  Now ask learners to draw the lines of symmetry on the shapes and compare with another learner.  *Did you find all the lines of symmetry?*  Learners will show they are **specialising (TWM.01)** when they are able to identify lines of symmetry in different shapes.  **Resources:**  Polygons | **Possible misconceptions:**  Some learners only see the prototypical images of shapes so do not necessarily recognise lines of symmetry in shapes that are presented in different orientations. |
| **3Gp.02** Sketch the reflection of a 2D shape in a horizontal or vertical mirror line, including where the mirror line is the edge of the shape. | Provide learners with a selection of 2D shapes and a mirror. For example:  Ask learners to sketch the reflection of the shapes in a mirror line parallel to one edge of the shape, where the edges of the shape are parallel or perpendicular to the mirror line. Then, ask them to select a different edge and repeat the activity.  Remind learners that equivalent points are the same (shortest) distance from the line of symmetry.  **Resources:**  Sheet with 2D shapes  Mirrors | **Possible misconceptions:**  Learners sometimes sketch the correct shape but not the correct distance from the mirror line.  Learners sometimes draw the correct shape but rotate or translate it rather than reflect it. |

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| Unit 3.3 Topic 3 Angles, direction and position |
| Outline of topic: |
| Learners will explore angles, specifically right angles. They will use angles and other positional language to describe and create position, direction and movement of shapes and objects. |
| Language: |
| **Key vocabulary:**  right angle, turn, direction, quarter turn, half turn, clockwise, anticlockwise  north, south, east, west  forwards, backwards  clockwise, anti-clockwise  **Key phrases:**  What is the difference between a half turn and a quarter turn?  How many right angles are there in a straight line?  We have turned … right angles. |
| Recommended prior knowledge: |
| * Understand that an angle is a description of a turn * Understand clockwise and anti-clockwise * Have knowledge of position and direction to describe movement |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Gg.10** Compare angles with a right angle. Recognise that a straight line is equivalent to two right angles or a half turn.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Give learners two drinking straws. Stick one drinking straw onto a piece of paper. Use the other drinking straw to answer the following questions.   * *Show me a right angle.* * *Show me another right angle in a different position.* * *Show me an angle that is not a right angle.* * *Make a right angle, and then continue to make another right angle. What do you notice? (You have now made a straight line or half a turn).* * *Make a quarter turn with your drinking straw. What do you notice? (It is the same as making a right angle).* * *Make half a turn with your drinking straw. How many right angles have you made?*   Learners will show they are **characterising (TWM.05)** when they identify the features of right angles.  **Resources:**  Drinking straws  Sticky tape or glue | **Possible misconceptions:**  Learners sometimes incorrectly think that a right angle can only consist of one turn, so they do not recognise that a straight line is two right angles. |
| **3Gp.01** Interpret and create descriptions of position, direction and movement, including reference to cardinal points.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Show learners an image of a compass showing the cardinal points: north, south, east and west.  In pairs, ask learners to plan a route on a piece of paper. Then, one learner gives instructions to the other learner to navigate the route, based on instructions such as:   * Go north 12 * Go west 7 * Go forwards 6 * Turn a right angle anti-clockwise * Go backwards 8   Learners will show they are **characterising (TWM.05)** when they identify and use a variety of positional language to plan their route. | **Possible misconceptions:**  Learners may not have enough vocabulary to use appropriate language to describe position. So, you may need to display the vocabulary required to assist learners. |

# Unit 3.4 Patterns, place value and rounding

| Learning objectives covered in Unit 3.4 and topic summary: | | 3.4 Topic 1  Making patterns | 3.4 Topic 2  Using place value and rounding | Thinking and Working Mathematically |
| --- | --- | --- | --- | --- |
| **3Nc.02** | Count on and count back in steps of constant size: 1-digit numbers, tens or hundreds, starting from any number (from 0 to 1000). | ✓ |  | **TWM.03 Conjecturing** |
| **3Nc.05** | Recognise and extend linear sequences, and describe the term-to-term rule. | ✓ |  | **TWM.02 Generalising** |
| **3Nc.06** | Extend spatial patterns formed from adding and subtracting a constant. | ✓ |  | **TWM.02 Generalising**  **TWM.04 Convincing** |
| **3Np.01** | Understand and explain that the value of each digit is determined by its position in that number (up to 3-digit numbers). |  | ✓ | **TWM.04 Convincing** |
| **3Np.02** | Use knowledge of place value to multiply whole numbers by 10. |  | ✓ | **TWM.04 Convincing** |
| **3Np.03** | Compose, decompose and regroup 3-digit numbers, using hundreds, tens and ones. |  | ✓ | **TWM.05 Characterising** |
| **3Np.05** | Round 3-digit numbers to the nearest 10 or 100. |  | ✓ | **TWM.04 Convincing** |

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| Unit 3.4 Topic 1 Making patterns |
| Outline of topic: |
| Learners will explore counting on and back with 1-digit numbers, tens and hundreds.  Learners will build on their understanding of pattern by exploring and developing spatial patterns. Learners will also begin to explore linear equations and develop an understanding of the term-to-term rule. |
| Language: |
| **Key vocabulary:**  place value, digit, position,  pattern, increase, decrease, first term, second term, term-to-term rule, constant  **Key phrases:**  What is the term-to-term rule?  What is the first term in this sequence? |
| Recommended prior knowledge: |
| * Counting forwards and backwards in steps of constant size up to 100 * Knowledge of numerical sequences |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
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| **3Nc.02** Count on and count back in steps of constant size: 1-digit numbers, tens or hundreds, starting from any number (from 0 to 1000).  **TWM.03 Conjecturing**  Forming mathematical questions or ideas | Explain to learners that you are all going to start at 7 and count up in 4s.  Count aloud, slowly, with everyone joining in: 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67, 71, 75, 79, 83, 87, 91, 95, 99 and stop.  Ask learners:  *What did you notice?*  Learners may have been thinking so much about counting that they did not notice anything, so ask them to repeat the task, this time writing the numbers, and see what they notice.  They may notice that:   * they did not count to exactly 100 * all the ones digits are odd * when the 10s digit is odd, the ones digit is 1, 5, 9; when the 10s digit is even, the ones digit is 3, 7.   Ask learners if they can write questions based on some of their observations. Then ask learners to form their own sequences and ask another learner to say what the sequence is and suggest at least one pattern that they identify in the sequence.  Ask learners:  *What will happen for this sequence with 3-digit numbers?*  *What will the sequence look like if you start at 6 instead of 7?*  Learners will show they are **conjecturing (TWM.03)** when they suggest ideas such as:   * The sequence will never get to exactly 200. * The sequence will include 523. | Purposeful counting is an essential skill in mathematics as it provides key building blocks to support calculation and other mathematical content.  Learnersoften find it difficult to count in odd numbers so include examples with odd numbers. Learners also find it difficult to count in fours. Encourage them to use strategies such as counting in twos, twice. |
| **3Nc.05** Recognise and extend linear sequences, and describe the term-to-term rule.  **TWM.02 Generalising**  Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria | Remind learners that a number pattern that increases (or decreases) by the same amount each time is called a linear sequence.  Write these number patterns on the board and ask learners what they notice:  2, 6, 10, 14 … (Answer: add 4 each time)  23, 18, 13, 8, … (Answer: subtract 5 each time)  Ask learners to draw a picture for each term to show what is changing each time. For example, learners could draw a pattern of squares or dots that increases each time. Ask learners to explain what is changing in their pattern each time. Encourage explanations that draw upon an increasing pattern that changes in the same way each time.  Each number in a pattern is called a term. So, in the number pattern 2, 6, 10, 14 … the first term is 2, the second term is 6, the third term is 10 and so on.  Ask learners to write down the rule and the next two terms in the number pattern 2, 5, 8, 11, 14 … (Answer: the rule is to add 3 each time, so the next numbers are 17, then 20)  Learners will show they are **generalising (TWM.02)** when they notice the rule and apply it consistently.  Ask learners to create their own number patterns (linear sequences) and then swap with a partner to solve. | **Possible misconceptions:**  Sometimes learners do not recognise the mathematical relationship between the visual appearance of the sequence and the rule. This relationship can be developed by linking directly to pictorial representation patterns such as a set of blocks which increase or decrease by the same quantity each time. |
| **3Nc.06** Extend spatial patterns formed from adding and subtracting a constant.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution  **TWM.02 Generalising**  Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria | Ask learners to solve the NRICH task: Sticky Triangles (<https://nrich.maths.org/88>).  If available, it would be helpful to have some matchsticks or drinking straws so that learners can make the triangles too.  Ask learners to predict how many matchsticks are needed to make the 3rd, 4th and 5th triangle. Ask learners to draw the representations to convince others that their prediction is correct. Learners will show they are **convincing (TWM.04)** when they draw representations of the growing patterns and explain to convince others of their solutions.  Ask learners if they can see a link between the number of small triangles and the total number of matchsticks used. Learners will show they are **generalising (TWM.02)** when they notice certain properties of the sequence.  **Resources:**  NRICH task  Matchsticks or drinking straws | **Possible misconceptions:**  Learners think that the constant can change.  Learners might not make the link between a pictorial representation of a growing pattern and its numerical rule. This can be developed using pictorial representations, |

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| Unit 3.4 Topic 2 Using place value and rounding |
| Outline of topic: |
| Learners will understand the value of each digit up to 3-digit numbers, and know that the position of a digit determines its value. They will use knowledge of place value to multiply numbers by ten and to compose, decompose and re-group 3-digit numbers.  Learners will round numbers to the nearest 10 or 100. |
| Language: |
| **Key vocabulary:**  place value, digit, hundreds, tens, ones, position  multiply, ten times bigger,  compose, decompose, re-group  round, nearest  **Key phrases:**  What do you notice about the digits?  Write … to the nearest …  Round … to the nearest … |
| Recommended prior knowledge: |
| * Understanding of place value for 2-digit numbers * Compose and decompose 2-digit numbers * Round 2-digit numbers |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
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| **3Np.01** Understand and explain that the value of each digit is determined by its position in that number (up to 3-digit numbers).  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners:  W*hat number does the digit 3 represent in 362? The 6? The 2?*  *What does each of the digits in the number 444 represent?*  Write the following numbers on the board and ask learners to find the missing numbers.  Ask learners to explain why.  642 = ? + 40 + 2  *Why does the digit 6 represent 600 and not 60 or 6?*  957 = 900 + ? + 7  *Why does the digit 5 represent 50 and not 500 or 5?*  398 = 300 + 90 + ?  *Why does the digit 8 represent 8 and not 800 or 8?*  Learners will show they are **convincing (TWM.04)** when they can justify the correct value of the digits, according to their position in the number sentence.  In pairs, give learners 0-9 digit cards. Ask them to choose three cards, and make the largest or smallest number that is odd or even.  Ask them to explain their strategy.  **Resources:**  0-9 digit cards | Positional place value is where a digit’s position in the number determines its value. A digit’s value is the digit multiplied by the value of its place.  **Possible misconceptions:**  Learners often decompose numbers correctly but then forget to add (regroup) the parts again. |
| **3Np.02** Use knowledge of place value to multiply whole numbers by 10.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Demonstrate, using counters, how a number grows ten times bigger when it is multiplied by ten.  Tell learners they are going to think about the calculation 3 x 10 = ?  Get three counters.  Now take another three counters, and another three and another three (until you have ten groups of three counters). If you add the ten groups of three counters together you will have 30 counters.  Using base 10 equipment (e.g. blocks) put the number 26 in a 100s, 10s and 1s table and show how the number changes when it is multiplied by 10. The 20 becomes 200 and the 6 becomes 60 as they move one place to the left and become ten times bigger.   |  |  |  | | --- | --- | --- | | 100s | 10s | 1s | |  | Column of 10 blocksColumn of 10 blocks | 1 of 6 blocks1 of 6 blocks1 of 6 blocks1 of 6 blocks1 of 6 blocks1 of 6 blocks | |  | 2 | 6 | | 2 | 6 | 0 | | 10x10 grid of 100 blocks  10x10 grid of 100 blocks | Column of 10 blocksColumn of 10 blocksColumn of 10 blocksColumn of 10 blocksColumn of 10 blocksColumn of 10 blocks |  |   Using blocks and a 100s, 10s, 1s table, ask learners to explore other numbers.  Finally ask learners to explain this pattern:  17 x 10 = 170  170 x 10 = 1700  1700 x 10 = 17000  Learners will show they are **convincing (TWM.04)** when they can explain how the pattern is growing each time.  **Resources:**  Counters  100s, 10s, 1s table  Base 10 blocks | Ensure learners understand that multiplying a number by 10 (or 100) shifts each of its digits one (or two) places to the left, it does not mean adding zeros.  **Possible misconceptions:**  Learners may talk about ‘adding a zero’ to a number when multiplying by 10. This is not helpful as it will lead to incorrect answers when multiplying decimals.  Learners sometimes incorrectly think that all multiplication makes numbers bigger, which is incorrect as multiplying by fractions or multiplying by less than 1 reduces the number. |
| **3Np.03** Compose, decompose and regroup 3-digit numbers, using hundreds, tens and ones.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Ask learners to represent these numbers using a *part whole model*, either using circles or bars: 236, 569, 145, 965, 369.  Provide the models for learners to use to compose and decompose the numbers.  Examples of the circle model:  One circle (236) connected to three circles (all blank)One circle (236) connected to three circles (200, 30 and 6)  Example of a bar model:   |  |  |  | | --- | --- | --- | |  | **569** |  | | **500** | **60** | **9** |   In pairs, ask learners to choose a 3-digit number but keep it hidden from their partner. Their partner then asks questions to guess the different parts of the number and put it back together.  For example, I might choose the number 738.  Learner 1: Does the number have the digit 7 in it?  Learner 2: Yes  Learner 1: Is the digit 7 greater than 70?  Learner 2: Yes  Learner 1: So that means part of the number is 700.  And so on, until the number is guessed correctly.  Then, ask learners to regroup the number.  Provide examples where learners need to regroup numbers. For example, the number 738 can be expressed as:   |  |  |  | | --- | --- | --- | | **100s** | **10s** | **1s** | | **7** | **3** | **8** | | **-** | **73** | **8** | | **-** | **-** | **738** |   Learners will show they are **characterising (TWM.05)** when they identify the different components of each number through their questioning and knowledge of place value. | A part whole model is a visual representation of how a number can be composed, decomposed or regrouped. Encourage learners to use mental strategies where possible.  Composing and decomposing should focus on every individual place value position: 100s (hundreds), 10s (tens) and 1s (ones).  100s10s1s = 100s + 10s + 1s  E.g. 712 = 700 + 10 + 2  Regrouping should focus on expressing a number in different ways to assist with calculations.  E.g. 712 can be expressed as:  71 tens and 2 ones  712 ones  7 hundreds, 1 ten and 2 ones  712 = 701 + 11  712 = 710 + 2  **Possible misconceptions:**  Learners may incorrectly believe that the number can only be represented in the order that the digits appear. For example, 400 + 80 + 6 = 486.  Ask learners:  *What does the digit 8 represent in this number?*  *Compare the numbers 486 and 468. What is the same and different?* |
| **3Np.05** Round 3-digit numbers to the nearest 10 or 100.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners to finish off these sentences:  633 is 630 rounded to the nearest ten because ….  837 is 840 rounded to the nearest ten because ….  433 is 400 rounded to the nearest hundred because …  857 is 900 rounded to the nearest hundred because …  45 rounded to the nearest hundred is 0 because ….  950 rounded to the nearest hundred is 1000 because….  Ask learners to round these distances from Penzance (England) to the nearest 100 miles and then to the nearest 10 miles.     |  |  | | --- | --- | | Aberdeen | 660 miles | | Edinburgh | 542 miles | | Fort William | 650 miles | | Kendal | 399 miles | | Leeds | 375 miles |   Learners will show they are **convincing (TWM.04)** when they can justify why their rounding is correct. Using a number line is helpful to show spatially how near numbers are to each other.  Show learners the grid below and ask:  *Can you find the mistakes?*   |  |  |  | | --- | --- | --- | |  | Rounded to the nearest 10 | Rounded to the nearest 100 | | 245 | 240 | 200 | | 621 | 620 | 700 | | 550 | 540 | 500 | | Rounding helps learners to make useful estimates.  **Possible misconceptions:**  Learners sometimes forget whether they should round up or down. Use a number line to show the distance between numbers.  Learners are sometimes confused when numbers less than 50 are rounded to the nearest 100 because they round down to 0. Learners are also sometimes confused when rounding numbers to the nearest 100 that result in the answer of 1000. |

# Unit 3.5 Measurement

| Learning objectives covered in Unit 3.5 and topic summary: | | | 3.5 Topic 1  Measuring length, mass, capacity and temperature | | Thinking and Working Mathematically | |
| --- | --- | --- | --- | --- | --- | --- |
| **3Gg.02** | Estimate and measure lengths in centimetres (cm), metres (m) and kilometres (km). Understand the relationship between units. | ✓ | | **TWM.07 Critiquing** | |
| **3Gg.03** | Understand that perimeter is the total distance around a 2D shape and can be calculated by adding lengths, and area is how much space a 2D shape occupies within its boundary. | ✓ | | **TWM.04 Convincing** | |
| **3Gg.04** | Draw lines, rectangles and squares. Estimate, measure and calculate the perimeter of a shape, using appropriate metric units, and area on a square grid. | ✓ | | **TWM.05 Characterising** | |
| **3Gg.06** | Estimate and measure the mass of objects in grams (g) and kilograms (kg). Understand the relationship between units. | ✓ | |  | |
| **3Gg.07** | Estimate and measure capacity in millilitres (m*l*) and litres (*l*), and understand their relationships. | ✓ | | **TWM.03 Conjecturing** | |
| **3Gg.11** | Use instruments that measure length, mass, capacity and temperature. | ✓ | | **TWM.03 Conjecturing**  **TWM.08 Improving** | |

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| Unit 3.5 Topic 1 Measuring length, mass, capacity and temperature |
| Outline of topic: |
| Learners will develop their understanding of the units of measurement, specifically centimetres, metres and kilometres and their symbols cm, m and km.  Learners will explore both perimeter and area and develop their skills of drawing lines, rectangles and squares.  Learners will have opportunities to use instruments that measure length, mass, capacity and temperature. They will develop their understanding through estimation and measurement. |
| Language: |
| **Key vocabulary:**  length, width, distance  centimetres (cm), metres (m), kilometres (km)  measure, ruler, metre stick, tape measure, metre stick, trundle wheel  mass, grams (g), kilograms (kg)  capacity, millilitres (m*l*), litres (*l*)  estimate, near to, more than, less than  **Key phrases:**  How long is …?  Can you estimate ….?  Which instrument should we use for measuring…?  Is this the perimeter or the area?  Is this lighter or heavier than …? |
| Recommended prior knowledge: |
| * Be able to estimate and measure length * Have an understanding of mass, capacity and temperature * Understand that mass is the quantity of matter in an object * Understand that capacity is the maximum amount an object can contain * Are familiar with equipment used to measure mass and capacity |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Gg.02** Estimate and measure lengths in centimetres (cm), metres (m) and kilometres (km). Understand the relationship between units.  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | Write these words on the board and ask learners which ones they recognise.  length, height, width, distance, centimetres (cm), metres (m), kilometres (km), measure, ruler, metre stick, tape measure, trundle wheel  Explain that these are all mathematical words associated with measuring length.  Ask learners to suggest what they might measure in:   * Centimetres (cm) * Metres (m) * Kilometres (km)   Share examples. Do all learners agree?  Ensure that learners know that 1 kilometre is 1000 times larger than 1 metre and that 1 metre is 100 times larger than 1 centimetre.  Learners will show they are **critiquing (TWM.07)** when they compare and evaluate the different responses given by their peers.  Give learners practical experiences of measuring length in centimetres and metres. Ask them to estimate first before measuring.  **Resources:**  Rulers  Tape measures  Metre sticks  Trundle wheels | At this stage learners have not been introduced to decimal numbers or numbers greater than 1000.  **Possible misconceptions:**  When measuring length, learners may not start at 0 when using measuring equipment. Instead they may incorrectly place the object at the start of the ruler at the ‘1’ division. |
| **3Gg.03** Understand that perimeter is the total distance around a 2D shape and can be calculated by adding lengths, and area is how much space a 2D shape occupies within its boundary.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Give learners rectangles with missing lengths and ask them to find the perimeter by measuring around the rectangle with a ruler. Give learners other shapes, such as triangles, with missing lengths and ask them to measure the lengths, and add them together to find the perimeter.  Next, give learners rectangles with the perimeter given but with no lengths. Ask them to suggest what the lengths of the rectangles could be.  *Is there more than one answer? Why?*  *What do you notice?*  *How can you find the perimeter?*  *How can you find the perimeter of the classroom?*  Challenge learners to draw a rectangle that has a perimeter of 24cm.  *How many different rectangles can you draw that have a perimeter of 24cm?*  Learners will show they are **convincing (TWM.04)** when they can justify why they think they have found all possible solutions.  Show learners this grid:   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   *How many squares are there inside the rectangle?*  Explain to learners that this is the area of the shape.  *What is the perimeter of this rectangle?*  Ask learners to draw another rectangle that has an area of 12cm2  *What is the perimeter of this rectangle?*  Challenge learners to draw as many different rectangles with an area of 12cm2  This activity can be extended by asking learners to draw a rectangle where the perimeter is larger than the area and another where the area is larger than the perimeter.  **Resources:**  Rulers | **Possible misconceptions:**  Learners may attempt to find the area of a shape by measuring the side lengths and adding them up. This would calculate the perimeter rather than the area.  Learners sometimes remember the definition of perimeter and area by the way they calculate it, so will often say that perimeter is adding the sides together and area is multiplying the sides together. It is important for learners to understand the definition of perimeter and area so that they can reason through problems. |
| **3Gg.04** Draw lines, rectangles and squares. Estimate, measure and calculate the perimeter of a shape, using appropriate metric units, and area on a square grid.  **TWM.05 Characterising**  Identifying and describing the mathematical properties of an object | Ask learners to draw lines that are 5cm long, 9cm long and 16cm long using a ruler. Encourage learners to check each other’s lines.  Challenge learners to draw a square (using squared paper) where each side is 14cm long.  *How long is the perimeter? What is the area of the square?*  Learners will show they are **characterising (TWM.05)** when they identify the features of a square (that all sides are equal length and all four angles are 90 degrees).  Challenge learners to draw a rectangle which has an area of 36cm2*.*  *What could the sides of this rectangle be?*  *What is the perimeter?*  Ask learners to choose a suitable unit of measure to estimate the area of:   * A postcard * A page of a book * A table * The length of the classroom   Ask leaners to measure the objects and compare their estimated value of area with the actual area.  **Resources:**  Rulers, Metre rules, tape measures  Squared paper | **Possible misconceptions:**  Learners may not start at zero when measuring using a ruler. |
| **3Gg.11** Use instruments that measure length, mass, capacity and temperature.  **TWM.03 Conjecturing**  Forming mathematical questions or ideas  **TWM.08 Improving**  Refining mathematical ideas or representations to develop a more effective approach or solution | Show learners a range of measuring equipment. For example:   * a ruler * a tape measure * a trundle wheel * a variety of scales * a thermometer * measuring cylinders.   Ask learners to explore things using each of the measuring equipment.  *Can you find something to measure with each piece of equipment?*  Encourage learners to make an estimate first, then measure, and finally compare their estimate with their measurement.  *How close was your estimate compared to the actual value?*  Learners will show they are **conjecturing (TWM.03)** when they suggest which piece of measuring equipment to use and then demonstrate they are **improving (TWM.08)** when they test this out and offer a more accurate conjecture.  Ask learners which measuring instrument they would choose to measure:   * the length of the classroom * a small book * a fence * the temperature of a glass of water * the mass of a bag of potatoes * a person * the capacity of a bottle * the mass of a ball of wool * the temperature in the classroom.   **Resources:**  Rulers  Tape measures  Trundle wheels  A variety of scales  Thermometers  Measuring cylinders | **Possible misconceptions:**  Learners are sometimes confused with the different units of measurement e.g. length is measured in centimetres, metres or kilometres; mass is measured in kg or g; capacity is measured in *l* or m*l*; and temperature is measured in degrees. |
| **3Gg.06** Estimate and measure the mass of objects in grams (g) and kilograms (kg). Understand the relationship between units. | Ensure learners know that 1kg is 1000 times heavier than 1g.  Give learners an object to hold that has a mass of 1kg.  Once they have experienced this, ask learners to suggest other objects that have a mass which is lighter or heavier than 1kg.  For example, if learners suggest a book is lighter than 1kg, ask them how much lighter is it? Is the book half as heavy? How many books would you need to have a total mass of 1kg?  Give learners opportunities to find objects that are more than 1kg and less than 1kg. Remind learners to estimate the mass of each object before weighing it.  This activity could be extended by giving learners the NRICH task: Money Measure (<https://nrich.maths.org/2417>).  **Resources:**  An object that has a mass of 1kg  NRICH task | **Possible misconceptions:**  Learners sometimes confuse the measurements of grams and kilograms. |
| **3Gg.07** Estimate and measure capacity in millilitres (m*l*) and litres (*l*), and understand their relationships.  **TWM.03 Conjecturing**  Forming mathematical questions or ideas | Give learners an object to hold that has a capacity of 1 litre.  Once they have experienced this, ask learners to suggest objects that have a capacity that is less than or more than 1 litre.  Ensure learners know that 1 litre is 1000 times larger than 1 millilitre.  Give learners a range of different measuring containers, cylinders and jugs. Ask learners to predict which container holds the most (largest capacity) and the least (smallest capacity).  Learners will show they are **conjecturing (TWM.03)** when they suggest the approximate capacity of each container.  Give learners this NRICH task to solve: Oh Harry! (<https://nrich.maths.org/5979>).  Watch the video clip from the NRICH task: Pouring Problem (https://nrich.maths.org/13664).  Ask learners:  *Are you surprised by the outcome?*  *Can you explain what is happening?*  **Resources:**  An object that has a capacity of 1 litre  A variety of containers, cylinders and jugs and some water  NRICH tasks | **Possible misconceptions:**  Learners sometimes confuse the measurements of millilitres and litres. |

# Unit 3.6 Multiplication and division

| Learning objectives covered in Unit 3.6 and topic summary: | | 3.6 Topic 1  Using multiplication and division facts | 3.6 Topic 2  Strategies for multiplication and division | Thinking and Working Mathematically |
| --- | --- | --- | --- | --- |
| **3Ni.05** | Understand and explain the relationship between multiplication and division. | ✓ |  | **TWM.07 Critiquing** |
| **3Ni.06** | Understand and explain the [commutative and distributive properties of multiplication](javascript:void(0)), and use these to simplify calculations. | ✓ |  | **TWM.04 Convincing** |
| **3Ni.07** | Know 1, 2, 3, 4, 5, 6, 8, 9 and 10 times tables. | ✓ |  | **TWM.07 Critiquing** |
| **3Ni.08** | Estimate and multiply whole numbers up to 100 by 2, 3, 4 and 5. |  | ✓ |  |
| **3Ni.09** | Estimate and divide whole numbers up to 100 by 2, 3, 4 and 5. |  | ✓ | **TWM.01 Specialising** |
| **3Ni.10** | Recognise multiples of 2, 5 and 10 (up to 1000). | ✓ |  | **TWM.02 Generalising**  **TWM.04 Convincing** |

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| Unit 3.6 Topic 1 Using multiplication and division facts |
| Outline of topic: |
| Learners will develop their understanding of the relationship between multiplication and division. They will explore the commutative and distributive laws of multiplication.  Learners will recognise multiples of 2, 5 and 10 and know their 1, 2, 3, 4, 5, 6, 8, 9 and 10 times tables. |
| Language: |
| **Key vocabulary:**  multiplication, multiply, division, divide  commutative, distributive, laws of arithmetic  multiples  **Key phrases:**  An example of the commutative law is …  An example of the distributive law is … |
| Recommended prior knowledge: |
| * Understand multiplication as repeated addition and an array * Understand division as repeated subtraction, sharing and grouping * Know the 2, 5 and 10 times tables |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Ni.05** Understand and explain the relationship between multiplication and division.  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | Show learners this array:   |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |   Ask learners to write multiplication and division sentences to describe this rectangle. For example, 3 × 9 = 27, 27 = 9 × 3 or 27 ÷ 3 = 9, 27 ÷ 9 = 3  Ask learners:  *What do you notice about the number sentences?*  Give learners other arrays and ask them to write multiplication and division sentences to describe them.  Learners will show they are **critiquing (TWM.07)** when they compare and contrast multiplication and division equations for the same representation. | Encourage learners to use mental strategies where possible, including knowledge of multiplication and division facts.  **Possible misconceptions:**  Learners sometimes do not see multiplication and division as inverse operations. |
| **3Ni.06** Understand and explain the [commutative and distributive properties of multiplication](javascript:void(0)), and use these to simplify calculations.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Explain that commutative property of multiplication means that some parts of a number sentence can be rearranged and will still provide the same answer (product).  Give learners some multiplication questions to complete. For example:  ? × 6 = 48  6 × ? = 48  Show learners this array, 12 × 4 = 48:   |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |   Explain that 12 × 4 is the same as 10 + 2 multiplied by 4 and that this is the same as the total of 10 multiplied by 4 and then 2 multiplied by 4.  Explain the distributive law.  Show learners the following array, which represents 3 × 4 = 12:   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  | |  |  |  |  |   This array is two lots of 3 × 2 which is the same as 6 + 6 = 12:   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  | |  |  |  |  |  |   This array is the one lot of 6 × 2 = 12:   |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  | |  |  | |  |  |   *3 × 4 = 6 × 2* *True or false?*  Learners will show they are **convincing (TWM.04**) when they can justify if this statement is true or false. | Encourage learners to use mental strategies where possible, including knowledge of multiplication facts.  Using the language of multiplicand × multiplier = product will enable learners to explain their thinking because they can name the different parts of the calculation.  **Possible misconceptions:**  Learners may apply the commutative and distributive laws incorrectly. Learners need to know that commutativity is related to rearranging part of the number sentence whereas the distributive rule involves splitting up the numbers. |
| **3Ni.07** Know 1, 2, 3, 4, 5, 6, 8, 9 and 10 times tables.  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | Choose a times table (e.g. 6 times table) and ask learners to count forwards from zero, so they say the multiples. Once they have counted to the tenth multiple, ask learners to count back down in the multiples to zero. You can use a counting stick, divided by lines into 10 sections, as a model for counting on and back. Repeat this several times for different times tables.  Next ask learners to draw an empty number line and ask them to show the jumps of the times table you are focusing on.  Finally use an array model to build up each table’s fact. For example:  6 × 1 = ☐☐☐☐☐☐  6 × 2 = ☐☐☐☐☐☐  ☐☐☐☐☐☐  6 × 3 = ☐☐☐☐☐☐  ☐☐☐☐☐☐  ☐☐☐☐☐☐  Repeat this for each of the times tables.  Ask learners to choose a multiplication fact from the times tables and build a fact list.  For example: ‘If I know 6 × 5 = 30 then I also know  30 = 5 × 6  50 × 6 = 300…’  Learners will show they are **critiquing (TWM.07)** when they compare and evaluate different approaches to solving times tables questions.  This activity can be extended by giving learners the NRICH task: Mystery Matrix (<https://nrich.maths.org/1070>).  **Resources:**  Counting stick  NRICH task | Being able to recall times tables facts is essential for learners to become effective at calculating.  **Possible misconceptions:**  Learners may be able to recall the answers to times table questions quickly, but may not be able to apply this information in other situations, e.g. they know that 4 × 5 = 20 but do not know that there is a relationship between this question and 40 x 5. |
| **3Ni.10** Recognise multiples of 2, 5 and 10 (up to 1000).  **TWM.02 Generalising**  Recognising an underlying pattern by identifying many examples that satisfy the same mathematical criteria  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Show learners this table, and ask them which numbers would they say when counting in 2s, 5s and 10s if they started from zero?   |  |  |  |  |  | | --- | --- | --- | --- | --- | | 705 | 55 | 102 | 10 | 185 | | 145 | 36 | 171 | 400 | 900 | | 60 | 98 | 275 | 222 | 1000 |   Ask learners:  *Do you notice anything about any of the numbers?*  For example, why did they not say the number 171 but they did say the number 145?  Learners will show they are **generalising (TWM.02)** when they recognise the properties and structure of multiplication tables. For example, all numbers with a 5 in the 1s place are multiples of 5.  Ask learners to create their own table where every number is a multiple of 2 and 5 and 10.  Ask learners to convince you how they know that the numbers in their tables are multiples of 2 and 5 and 10.  Learners will be **convincing (TWM.04)** when they are able to show that numbers with an even number other than 0 in the 1s place are multiples of 2, numbers with a 5 in the 1s place are a multiple of 5 and numbers with a 0 in the 1s place are multiples of 2, 5 and 10. |  |

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| Unit 3.6 Topic 2 Strategies for multiplication and division |
| Outline of topic: |
| Learners will estimate and then multiply and divide whole numbers up to 100 by 2, 3, 4 and 5. |
| Language: |
| **Key vocabulary:**  multiplication, multiply  division, divide  estimate, near to  **Key phrases:**  Can you estimate ….?  Does that seem reasonable? |
| Recommended prior knowledge: |
| * Understand multiplication as repeated addition and as an array * Understand division as repeated subtraction, sharing and grouping |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Ni.08** Estimate and multiply whole numbers up to 100 by 2, 3, 4 and 5. | Ask learners to choose any number from the grid, and multiply it by 2, 3, 4 or 5.  *Can you estimate what the answer will be first?*   |  |  |  |  | | --- | --- | --- | --- | | 25 | 17 | 19 | 80 | | 12 | 98 | 32 | 13 | | 78 | 3 | 44 | 34 | | 50 | 64 | 56 | 16 |   In pairs, ask learners to choose two 1-digit cards each (from a 0-9 pack) and make a two-digit number that they can then multiply by either 2, 3, 4 or 5. For each calculation, choose a different number for learners to try and find. For example, can you find a number greater than 50, smaller than 25, an odd number, even number?  **Resources:**  0-9 digit cards | Encourage learners to use mental strategies where possible, including knowledge of multiplication facts.  Using the language of multiplicand × multiplier = product will enable learners to explain their thinking because they can name the different parts of the calculation.  **Possible misconceptions:**  Learners often try to calculate the answer immediately rather than offering an estimate first. |
| **3Ni.09** Estimate and divide whole numbers up to 100 by 2, 3, 4 and 5.  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | Ask learners to choose any number from the grid, and divide it by 2, 3, 4 or 5:   |  |  |  |  | | --- | --- | --- | --- | | 25 | 18 | 18 | 80 | | 12 | 95 | 32 | 38 | | 78 | 5 | 44 | 34 | | 50 | 64 | 56 | 33 |   Ask learners:  *Can you estimate what the answer will be first?*  *Will you be able to divide all the numbers by 2, 3, 4 and 5?*  Now show learners a bar model that has been used to solve a problem, such as the example below:   |  |  |  | | --- | --- | --- | |  | ☐☐☐☐☐☐☐☐☐  ☐☐☐☐☐☐☐☐☐ |  | | ☐☐☐  ☐☐☐ | ☐☐☐  ☐☐☐ | ☐☐☐  ☐☐☐ |   *What could the question have been?* (Answer: What is 18 divided by 3?)  Ask learners to create their own bar model representations for multiplication and division questions, and then swap with a partner calculate the answer.  Learners will show they are **specialising (TWM.01)** by testing examples to see if they satisfy the bar model representation. | Encourage learners to use mental strategies where possible, including knowledge of division facts.  **Possible misconceptions:**  Learners often try to find the answer immediately rather than offer an estimate first. |

# Unit 3.7 Fractions

| Learning objectives covered in Unit 3.7 and topic summary: | | 3.7 Topic 1  Understanding fractions | 3.7 Topic 2  Comparing fractions | 3.7 Topic 3  Calculating with fractions | Thinking and Working Mathematically |
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| **3Nf.01** | Understand and explain that fractions are several equal parts of an object or shape and all the parts, taken together, equal one whole. | ✓ |  |  | **TWM.04 Convincing** |
| **3Nf.02** | Understand that the relationship between the whole and the parts depends on the relative size of each, regardless of their shape or orientation. | ✓ |  |  | **TWM.04 Convincing** |
| **3Nf.03** | Understand and explain that fractions can describe equal parts of a quantity or set of objects. | ✓ |  |  | **TWM.07 Critiquing** |
| **3Nf.04** | Understand that a fraction can be represented as a division of the numerator by the denominator (half, quarter and three-quarters). | ✓ |  |  |  |
| **3Nf.05** | Understand that fractions (half, quarter, three-quarters, third and tenth) can act as operators. |  |  | ✓ | **TWM.04 Convincing** |
| **3Nf.06** | Recognise that two fractions can have an equivalent value (halves, quarters, fifths and tenths). |  | ✓ |  | **TWM.04 Convincing**  **TWM.07 Critiquing**  **TWM.08 Improving** |
| **3Nf.07** | Estimate, add and subtract fractions with the same denominator (within one whole). |  |  | ✓ | **TWM.04 Convincing**  **TWM.08 Improving** |
| **3Nf.08** | Use knowledge of equivalence to compare and order unit fractions and fractions with the same denominator, using the symbols =, > and <. |  | ✓ |  | **TWM.03 Conjecturing**  **TWM.04 Convincing**  **TWM.08 Improving** |

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| Unit 3.7 Topic 1 Understanding fractions |
| Outline of topic: |
| Learners will develop their understanding of fractions by knowing that they represent several parts of an object or shape. They will use this knowledge to understand the relationship between the whole and the parts.    They will understand that fractions can describe equal parts of a quantity or set of objects and that a fraction can be represented as a division of the numerator by the denominator. |
| Language: |
| **Key vocabulary:**  numerator, denominator, division, dividing line  whole, parts, equal, unequal  half, quarter, three-quarters  **Key phrases:**  What does this fraction represent?  How many equal parts are there in …?  A quarter is divided into …parts? |
| Recommended prior knowledge: |
| * Understand that an object or shape can be divided into two or four equal or unequal parts * Understand that a half and a quarter can describe two or four equal parts of a quantity or set of objects * Understand that one half and one quarter can be interpreted as division |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nf.01** Understand and explain that fractions are several equal parts of an object or shape and all the parts, taken together, equal one whole.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Show learners a picture of 8 buttons, with a circle around two of them and ask: *What fraction of the buttons is in the circle?*  *What fraction of the buttons is not in the circle?*  *How do you know?*  Show that the two parts add up to a whole so the fraction that is in the circle and the fraction that is not in the circle add up to one whole.  Give each learner a piece of A4 paper and ask them to halve it in as many different ways as they can. Often learners may start by halving the paper vertically or horizontally, but encourage them to be creative. What about diagonally? What about into quarters, where two quarters represent a half?  *How do you know they are halves?*  You can also try different sizes of paper and different fractions.  Now ask learners to colour a regular hexagon (divided into six equal sections) so that is yellow and is red.  Regular hexagon divided into six equal sections, three yellow, two red and one white  Learners will show they are **convincing (TWM.04**) when they can justify why they have coloured in the hexagon accurately.  Try this exercise with other shapes divided into different numbers of sections such as 12, which has factors of 2, 3 and 4. For example, a rectangle with 12 sections or an octagon with 8 sections. Ask learners to draw their own shapes such as rectangles using squared paper.  *How do you know you have coloured in the shape correctly?*  **Resources:**  A4 paper, paper of different sizes Squared paper  Regular hexagons divided into 6 parts, other shapes divided into equal parts | It is important when working with fractions to remind learners about the whole-part relationships.  **Possible misconceptions:**  Learners may see an object divided into two unequal parts and consider them to be halves. |
| **3Nf.02** Understand that the relationship between the whole and the parts depends on the relative size of each, regardless of their shape or orientation.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Draw two partially completed number lines on the board:    0  0  The first line starts at 0 and is marked on the line but the rest of the line is hidden. The second line also starts at 0 and shows marked in the same place as the , but again the rest of the line is hidden.  *Which number line is longer?*  *Explain how you know?*  Learners will show they are **convincing (TWM.04**) when they can justify and explain the hidden part of the number line.  Now give learners the NRICH task to solve: Fair Feast (<https://nrich.maths.org/2361>).  **Resources:**  NRICH task | **Possible misconceptions:**  Learners may believe that any object that is divided into 10 parts has been split into tenths, even when the 10 parts are not of equal size. |
| **3Nf.03** Understand and explain that fractions can describe equal parts of a quantity or set of objects.  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages | Ask learners to draw different pictures (circles, bars, shapes) to show that:   * A half is divided into two equal parts * A quarter is divided into four equal parts * A third is divided into three equal parts * A tenth is divided into ten equal parts   Learners will show they are **critiquing (TWM.07)** when they can identify possible advantages and disadvantages for using one representation over another.  Show learners a picture of 12 balls, with a circle around four of them and ask: *What fraction of the balls is in the circle?*  *What fraction of the balls is not in the circle?*  *How do you know?*  Encourage learners to recognise what is not , , or when the parts are not equal.  For example, ask learners to explain why this shape is not divided into thirds:   |  |  |  | | --- | --- | --- | |  |  |  | | It is important when working with fractions to remind learners about the whole-part relationships.  **Possible misconceptions:**  Learners may not understand that a half can be represented in different ways, such as of a shape. |
| **3Nf.04** Understand that a fraction can be represented as a division of the numerator by the denominator (half, quarter and three-quarters). | Write the following fractions on the board: , ,  Ask learners to choose a fraction from the board and draw a bar model to show that the numerator is divided by the denominator.  For example:   |  |  | | --- | --- | | One whole | | | One half | One half |   Give learners several word problems to solve. For example, there are 8 sweets. You take 2 of them. What fraction of the sweets did you take?  2 out of 8 = 2 =  Show learners that fractions can also be written as a division sentence:  1 ÷ 4 = . | Ensure learners know that the denominator is the bottom number and shows in how many equal parts the whole was divided into. The numerator is the top number and shows how many parts we are considering. For example:  **Possible misconceptions:**  Learners may not realise that the line between the numerator and denominator (vinculum) actually represents division. |

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| Unit 3.7 Topic 2 Comparing fractions |
| Outline of topic: |
| Learners will develop their understanding of equivalent fractions and use this to compare and order unit fractions and fractions with the same denominator. |
| Language: |
| **Key vocabulary:**  fractions, parts, whole, equivalent, not equivalent  numerator, denominator  equals, more than, less than, compare, order  whole, half, quarter, three-quarters, fifths, tenths  **Key phrases:**  Is this an equivalent fraction?  Is … more than or less than …?  Can you show me a fraction that is not equivalent to …? |
| Recommended prior knowledge: |
| * Be able to recognise and visualise a quarter, half, two quarters and three quarters. |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nf.06** Recognise that two fractions can have an equivalent value (halves, quarters, fifths and tenths).  **TWM.07 Critiquing**  Comparing and evaluating mathematical ideas, representations or solutions to identify advantages and disadvantages  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution  **TWM.08 Improving**  Refining mathematical ideas or representations to develop a more effective approach or solution | Show learners the information below:  *Jamila says that her diagram shows that*   |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  |   Ask learners:   * *Do you agree or disagree with Jamila?* * *Explain why Jamila might think she is correct.* * *Explain why Jamila is wrong.* * *Can you improve Jamila’s diagrams?*   Learners will show they are **critiquing (TWM.07)** when they evaluate Jamila’s diagrams and identify why her representations are incorrect. They will show they are **convincing (TWM.04)** when they challenge Jamila’s ideas and explain why she is wrong. They will show they are **improving (TWM.08)** when they suggest a more accurate representation of the fractions.  Now show learners the following two diagrams which are divided into equal parts. The shaded part of the first diagram represents and the second one .   |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  |   Ask learners if they can explain why is the same size as .  Ask them to draw other fractions that have an equivalent value. | **Possible misconceptions:**  Jamila’s work shows a misconception because in the top diagram the three parts are not equal in size. So, both diagrams represent . |
| **3Nf.08** Use knowledge of equivalence to compare and order unit fractions and fractions with the same denominator, using the symbols =, > and <.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution  **TWM.08 Improving**  Refining mathematical ideas or representations to develop a more effective approach or solution  **TWM.03 Conjecturing**  Forming mathematical questions or ideas  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask which is larger, or ?  Ask learners to draw a picture, use a number line or bar model to compare these fractions. Encourage learners to use a fraction wall, for example:   |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | |  |  |  |  |  |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |   Ask learners which of these fractions are greater than one half:  , ,  *Can you use the symbols > or < to put the equations in the correct position?*  For example: >  Yuri says that is less than .  *Is he correct?*  *Can you prove it?*  Learners will show they are **convincing (TWM.04)** when they challenge Yuri’s idea and explain why he is incorrect. They will show they are **improving (TWM.08)** when they suggest another fraction that correctly fits Yuri’s statement.  Repeat for other fractions. Ask learners to put their fractions in order of size and show them as one picture on a fraction wall.  *What do you notice?*  Encourage learners to share their thinking and conjecture which fractions might be in between those they have drawn. Learners will be **conjecturing (TWM.03)** when they suggest fractions in between those they have drawn and **convincing (TWM.04)** when they can convince others of how they know. | **Possible misconceptions:**  Learners may lack understanding of the relative size of fractions and so struggle to compare equivalent fractions accurately. Relate to diagrams that represent relative sizes of fractions such as the fraction wall.  Remind learners of the strategies for remembering the symbols < and >, such as they are like arrows that points to the smaller number. |

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| Unit 3.7 Topic 3 Calculating with fractions |
| Outline of topic: |
| Learners will extend their knowledge of fractions to understand that fractions can act as operators in mathematical problems.  Learners will explore estimating and then adding and subtracting fractions with the same denominator. |
| Language: |
| **Key vocabulary:**  fractions, parts, whole  numerator, denominator  whole, half, quarter, three-quarters, third, tenths  **Key phrases:**  What is of …?  Which fraction is equal to ...? |
| Recommended prior knowledge: |
| * Be able to recognise and visualise a quarter, half, two quarters and three quarters * Understand that one half and one quarter can be interpreted as division |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Nf.05** Understand that fractions (half, quarter, three-quarters, third and tenth) can act as operators.  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution | Ask learners to find half of each number from 1 to 30.  Model how to record the number sentence:  of 1 =  of 2 = 1  of 3 = 1  Challenge learners by asking questions such as:   * *What is of 20? Will the answer be greater than or less than 20?* * *What is of 20? Will the answer be greater than or less than 20?* * *What is of 18? Will the answer be greater than or less than 18?*   Learners will show they are **convincing (TWM.04)** when they can offer answers to these questions and explain how they know. | **Possible misconceptions:**  Learners may only see fractions as the result of a division and not as an operator.  Learners may see fractions as part of a shape but also need to know that it can be a part of a quantity. |
| **3Nf.07** Estimate, add and subtract fractions with the same denominator (within one whole).  **TWM.04 Convincing**  Presenting evidence to *justify or challenge* a mathematical idea or solution  **TWM.08 Improving**  Refining mathematical ideas or representations to develop a more effective approach or solution | Give learners fraction strips representing a whole, halves, quarters, thirds and tenths.  *Can you put them in order of their size using a fraction wall?*   |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | one whole | | | | | | | | | | | | | | |  | | | | | | |  | | | | | | | |  | | | | |  | | | |  | | | | | |  | | |  | | | |  | | | |  | | | |  |  |  | |  | |  |  |  | |  | |  |  |   Ask learners to solve the following questions using their fractions strips.   * + * - * + * - * + * –   *Jamillah has incorrectly written some number sentences. Explain her mistakes.*   * + = 1 whole * - =   Encourage learners to use their fraction strips to work out the answer.  Learners will show they are **convincing (TWM.04)** when they able to find the mistakes and offer reasons why, and **improving (TWM.08)** when they can suggest how to correct the equation.  **Resources:**  Fraction strips | **Possible misconceptions:**  Learners may add both the numerator and denominators. It is important that the learners understand they are counting how many of a given unit fractions so they do not add both numerators and denominators. So for example  ¼ + ¼ is adding quarters which makes 2/4 not 2/8. |

# Unit 3.8 Statistical methods and chance

| Learning objectives covered in Unit 3.8 and topic summary: | | 3.8 Topic 1  The statistical cycle | 3.8 Topic 2  Chance | Thinking and Working Mathematically |
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| **3Ss.01** | Conduct an investigation to answer non-statistical and statistical questions (categorical and discrete data). | ✓ |  |  |
| **3Ss.02** | Record, organise and represent categorical and discrete data. Choose and explain which representation to use in a given situation:   * Venn and Carroll diagrams * tally charts and frequency tables * pictograms and bar charts. | ✓ |  | **TWM.06 Classifying** |
| **3Ss.03** | Interpret data, identifying similarities and variations, within data sets, to answer non-statistical and statistical questions and discuss conclusions. | ✓ |  | **TWM.01 Specialising** |
| **3Sp.01** | Use familiar language associated with chance to describe events, including ‘it will happen’, ‘it will not happen’, ‘it might happen’. |  | ✓ | **TWM.01 Specialising TWM.06 Classifying** |
| **3Sp.02** | Conduct chance experiments, and present and describe the results. |  | ✓ | **TWM.03 Conjecturing** |

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| Unit 3.8 Topic 1 The statistical cycle |
| Outline of topic: |
| Learners will decide on a problem or question that can be investigated by collecting and analysing data. |
| Language: |
| **Key vocabulary:**  investigation, problem, questions, conclusions  data, non-statistical, statistical, categorical, discrete  Venn diagram, Carroll diagram, tally chart, frequency table, pictogram, bar chart  **Key phrases:**  What questions shall we ask?  How shall we represent this data? |
| Recommended prior knowledge: |
| * Experience of conducting an investigation * Lists, tables, tally charts, Venn diagrams, Carroll diagrams, pictograms, block graphs * Interpreting data and drawing conclusions |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Ss.01** Conduct an investigation to answer non-statistical and statistical questions (categorical and discrete data). | Ask learners for some topics to conduct a class investigation. Discuss the difference between categorical and discrete data. Can they suggest an investigation for both?  For example, learners might suggest finding out favourite sports, favourite animals or favourite films for categorical data and how many siblings they have, how many books they have read this month or the results of rolling two dice for discrete data.  Learners can collect data in different ways. For example, they could ask all learners to vote or place different counters in a jar to represent their favourite thing.  Once the class has decided on both a categorical data question and a discrete data question, collect the information for both investigations and then ask learners to consider how they will present the data.  Remember to phrase the investigation as a question so that the data can provide the answer.  For example: *What are learners’ favourite sports?* (football, swimming, golf)  You can also pose a question for learners to investigate and solve.  For example: *It is more likely for someone to turn over an odd number from a pack of 0-9 digit cards. Is this true?*  **Resources:**  Dice  0-9 digit cards | Categorical data refers to characteristics such as colour, names, personal preferences etc.  Discrete data refers to data that can be counted and has a finite number of possible values in a given range, such as number of siblings, how many books they have read this month etc.  **Possible misconceptions:**  Learners may not gather sufficient data.  Learners conduct simple statistics investigations as part of a four-part statistical enquiry cycle:  Statistics enquiry cycle: 1 Specify the problem and plan; 2 Record, organise and represent data; 3 Interpret data; 4 Discuss data and check predictions |
| **3Ss.02** Record, organise and represent categorical and discrete data. Choose and explain which representation to use in a given situation:   * Venn and Carroll diagrams * tally charts and frequency tables * pictograms and bar charts.   **TWM.06 Classifying**  Organising objects into groups according to their mathematical properties | Take learners on a nature walk. They might decide to collect information on different plants, flowers, insects etc. How many of these did you see in our play area?  Ask them to record their results first by using a tally chart and frequency table.  Then ask them to show their data in both a Venn diagram and a pictogram.  When they have finished ask them to decide which representation is more useful to display their information.  Learners will show they are **classifying** **(TWM.06)** when they sort the information.  Discuss with learners how the way we represent data often depends on the question we are asking.  *Hassan says that Venn diagrams and Carroll diagrams are more useful for presenting information than pictograms or bar charts. Do you agree?*  Ask learners to talk to a partner and decide if they agree with Hassan. | **Possible misconceptions:**  Learners think that all data is valid and accurate.  Learners conduct simple statistics investigations as part of a four-part statistical enquiry cycle. In the second part of the cycle, learners record, organise and represent data.  Statistics enquiry cycle: 1 Specify the problem and plan; 2 Record, organise and represent data; 3 Interpret data; 4 Discuss data and check predictions |
| **3Ss.03** Interpret data, identifying similarities and variations, within data sets, to answer non-statistical and statistical questions and discuss conclusions.  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria | This Carroll diagram records how some of the whole numbers from 20 to 39 were sorted.  Ask learners to add in the numbers 24 and 35.   |  |  |  | | --- | --- | --- | |  | Odd | Not odd | | Numbers that have 3 tens | 37  31 | 38  30 | | Numbers that do not have 3 tens | 23  25 | 26  20 |   *Will there always be an equal number of odd and even numbers in each box?*  *Are there any numbers between 20 and 39 that cannot be placed in any of the boxes?*  Learners will show they are **specialising (TWM.01)** when they identify examples of events to satisfy each of the criteria.  Show learners this table which shows the entrance fees to a museum over a number of years:   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Year | 2003 | 2006 | 2009 | 2012 | 2015 | | Entrance fee | £4.50 | £5.10 | £5.70 | £7.30 | £7.90 |   Ask learners:  *How might you represent this data?*  Ask learners to consider different representations e.g. Venn and Carroll diagrams, tally charts and frequency tables, pictograms and bar charts.  Ask learners*:*   * *Can you predict what the entrance fee might be in 2018? 2021?* * *Which representation will best assist you to answer the question?* * *What might the entrance fee have been in 2004? 2008? 2013?* * *In 2014 could the entrance fee have been £7.40?* * *What other information can you determine from this data?* | **Possible misconceptions:**  Learners often think that any amount of data is sufficient to answer a question.  Learners conduct simple statistics investigations as part of a four-part statistical enquiry cycle. In the third part of the cycle, learners interpret data and in the fourth part they discuss the data and check their predictions.  Statistics enquiry cycle: 1 Specify the problem and plan; 2 Record, organise and represent data; 3 Interpret data; 4 Discuss data and check predictions |

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| Unit 3.8 Topic 2 Chance |
| Outline of topic: |
| Learners will use the language associated with chance. They will be able to describe events including ‘it will happen’, ‘it will not happen’ and ‘it might happen’. |
| Language: |
| **Key vocabulary:**  chance, good chance, poor chance, no chance, risk, doubt  **Key phrases:**  Is it likely? Is it unlikely?  Can you be certain of …? |
| Recommended prior knowledge: |
| * Use familiar language associated with chance * Experience of planning chance experiments |

| Learning objectives | Suggested teaching activities and resources | Mental strategies, possible misconceptions and comments |
| --- | --- | --- |
| **3Sp.01** Use familiar language associated with chance to describe events, including ‘it will happen’, ‘it will not happen’, ‘it might happen’.  **TWM.01 Specialising**  Choosing *an example* and checking to see if it satisfies or does not satisfy specific mathematical criteria  **TWM.06 Classifying**  Organising objects into groups according to their mathematical properties | Ask learners to think of events that will definitely happen, definitely will not happen or might happen and to write them on individual pieces of paper. Learners should try to think of at least two examples for each.  Learners will show they are **specialising (TWM.01)** when they identify examples of events to satisfy each of the criteria.  Then ask learners to swap their pieces of paper with another learner and ask them to organise their partner’s events into ‘definitely will happen’, ‘definitely will not happen’ or ‘might happen’.  Learners will show they are **classifying** **(TWM.06)** when they correctly sort the events.  Then ask learners to compare their answers with what their partner and to discuss if they disagree. | **Possible misconceptions:**  Learners may incorrectly think that each outcome of every event is equally likely. |
| **3Sp.02** Conduct chance experiments, and present and describe the results.  **TWM.03 Conjecturing**  Forming mathematical questions or ideas | Give learners a coin and ask them to toss it in the air and observe if it lands on its head or tail. Take feedback from several learners. Are their results the same or different?  Ask learners:  *If the coin is tossed into the air again, do you think the coin is more likely to land on its head, rather than its tail?*  Learners will show they are **conjecturing (TWM.03)** when they suggest ideas as to the possible results of the experiment.  Ask learners to work in pairs and record the results on paper.  *In 20 tosses, did heads and tails each come up 10 times?*  *What happens if you combine your results with another pair?*  *How many times would you have to repeat the experiment to have a fair chance of tossing the coin equally to land on heads and tails?*  **Resources:**  Coins | **Possible misconceptions:**  Learners may think that if you have tossed a head several times then on the next toss it will definitely be a tail. Explain to learners that each toss of a coin is separate and has an equally likely chance of a head or a tail. The coin does not ‘remember’ the result of the previous toss. |

# Sample lesson 1

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| CLASS: | |
| DATE: | |
| **Learning objectives** | **3Nf.08** Use knowledge of equivalence to compare and order fractions with the same denominator. |
| **Lesson focus /**  **success criteria** | Learners can order and compare fractions with the same denominator.   * I can compare and order fractions with the same denominator. * I can use knowledge of equivalence to help me compare fractions. |
| **Prior knowledge /**  **previous learning** | Fractions are numbers.  The denominator determines the number of equal parts the whole is divided into and the numerator shows how many of these equal parts are represented by the fraction. |

**Plan**

| **Lesson** | **Planned activities** | **Notes** |
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| **Introduction** | Show learners the learning objectives and lesson focus and agree the success criteria:   * I can compare and order fractions with the same denominator. * I can use knowledge of equivalence to help me compare fractions.   Remind learners that fractions are numbers, but it is important to know how they compare to other fractions and how we can order them by their relative size.  Ask learners to consider this problem.  *Mia and her three friends have shared a box of counters. Sam has , Lucia has , Ali has and Mia has . Who has the most counters and who has the least?*  *How will Mia order these fractions from the smallest to largest?*  *, , ,*  Ask learners:   * *Why might Mia find this difficult?* * *Which fraction do you think Mia would like to change to make this easier for her?* * *Describe a different fraction that Mia might like to use instead.*   Agree that it is easier to order fractions when they all have the same denominator. This is because when they have the same denominator it means we are comparing the same unit size e.g. eighths. Now ask learners to consider this set of fractions:  , , , ,  Ask learners:   * *What do you notice this time?* * *What facts could you use to help you sort them from smallest to largest?*   Establish that we can order fractions that do not have the same denominator and that using knowledge of equivalence ( = in this case) will help us. |  |
| **Main activities** | In pairs, give learners a set of fraction cards. Ask learners to describe each card using the language:   * *There are …. equal parts and I have …. of them. I have ….*   *(the fraction).*   * *There are ten equal parts and I have three of them. I have .*  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | |  |  |  |  |  | |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | |  |  |  |  | |  |  |  |  |   Next, ask each pair to sort the cards into groups that share the same  denominator. Compare and order the fractions in each group. Write the  fractions in each group in order.  This activity can be extended by providing more fractions cards with different denominators.  Finally set learners this challenge:  Mia collects all the fraction cards representing eighths.  Tom collects the fraction cards representing tenths.  Peter does not collect fraction cards with eighths or tenths but he does collect other fractions.  Mia has five different fraction cards.  Tom has three different fraction cards.  Peter has three different fraction cards.  Create a set of fraction cards for Mia and write them in order.  Create a set of fraction cards for Tom and write them in order.  Now find three different fractions for Peter and then find three different fractions that can be used in Mia’s set and three different fractions that can be used in Tom’s set of fractions. | **Resources:**  Fraction cards |
| **End/Close/ Reflection/ Summary** | Ask learners:   * *Are these fractions in order from smallest to largest? , , .* * *How do you know?*   Remind learners that while it is easier to compare and order fractions with the same denominator, they can also use their knowledge of equivalence to help them sort fractions in order of size.  Revisit the learning objectives and success criteria. Ask learners to explain whether they have met the success criteria and if they have any questions or comments. |  |

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| **Reflection Use the space below to reflect on your lesson. Answer the most relevant questions for your lesson.** |
| *Were the learning objectives and lesson focus realistic? What did the learners learn today? What was the learning atmosphere like? What changes did I make from my plan and why?*  *If I taught this lesson again, what would I change?*  *What two things really went well (consider both teaching and learning)?*  *What two things would have improved the lesson (consider both teaching and learning)?*  *What have I learned from this lesson about the class or individuals that will inform my next lesson?* |
| **Next steps**  **What will I teach next based on learners’ understanding of this lesson?** |

# Sample lesson 2

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| --- | --- |
| CLASS: | |
| DATE: | |
| **Learning objectives** | **3Gg.03** Understand that perimeter is the total distance around a 2D shape and can be calculated by adding lengths. |
| **Lesson focus /**  **success criteria** | Learners understand what perimeter is and are able to calculate the perimeter of 2D regular and irregular polygons.   * I know what perimeter is. * I can find the perimeter of 2D shapes. |
| **Prior knowledge /**  **previous learning** | Understanding of length and distance.  Knowledge of cm and m. |

**Plan**

| **Lesson** | **Planned activities** | **Notes** |
| --- | --- | --- |
| **Introduction** | Show learners the learning objectives and lesson focus and agree the success criteria:   * I know what perimeter is. * I can find the perimeter of 2D shapes.   Give learners a card with two shapes drawn on it e.g. a blue equilateral triangle and a red square. (Make sure the shapes are drawn accurately and the sides of the shapes are easy to measure, e.g. each side of the triangle could be 5cm and each side of the square could be 3cm.)  Explain to learners that Mia is making shapes out of wool or string to stick onto card.  Ask learners:  *How much wool will Mia need to make each shape?*  Give learners the opportunity to solve this practically using wool or string.  *Has Mia used more or less wool to make the triangle or the square?*  Explain that the distance around all the sides of a shape is called the ‘perimeter’.  Show learners that they can also measure each side of a shape with a ruler, then they can add all the lengths of the sides together to find the perimeter.  Demonstrate this practically in front of the class, and establish that the triangle has a perimeter of 15cm and the square has a perimeter of 12cm. | **Resources:**  Shape cards  Wool  Ruler |
| **Main activities** | Provide learners with a sheet of different regular and irregular polygons drawn on. For example:  Ask learners to measure each side of one shape, add them all together and calculate the perimeter. Ask learners to record their calculations as a number sentence, e.g.  3cm + 4cm + 3cm = 10cm.  Remind learners to include the units. Ask learners to do this for all of the shapes.  Ask learners to check each other’s answers and then discuss as a class.  Ask learners:  *Do you need to measure all of the sides of each shape or do you know that some of them are the same length? How do you know?*  (For example, any regular polygon has sides of the same length.)  Now ask learners to draw two or three shapes and calculate the perimeter. Next, ask them to give these shapes, but not the perimeter, to a partner and challenge them to find the perimeter. Ask learners to compare their answers. Are they the same or different?  Finally inform learners that you are thinking of a shape that has a perimeter of 14cm.  *Can you draw a shape that has a perimeter of 14cm?*  *How many different shapes can you draw with a perimeter of 14cm?*  This activity can be extended by asking learners to estimate and then calculate the perimeter of objects around the classroom. Remind learners that they should use appropriate units of measure e.g. cm, m or km. | **Resources:**  Polygon worksheet  It is more important that learners begin to understand that perimeter is the distance around the outside of a shape, rather than being able to measure accurately. |
| **End/Close/ Reflection/ Summary** | Inform learners that you have calculated the perimeter of a square and a triangle and both have a perimeter of 12cm. Ask learners:  *Is this possible? What might the shapes look like?*  Revisit the learning objectives and success criteria. Ask learners to explain what perimeter is and how you can calculate it. |  |

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| **Reflection Use the space below to reflect on your lesson. Answer the most relevant questions for your lesson.** |
| *Were the learning objectives and lesson focus realistic? What did the learners learn today? What was the learning atmosphere like? What changes did I make from my plan and why?*  *If I taught this lesson again, what would I change?*  *What two things really went well (consider both teaching and learning)?*  *What two things would have improved the lesson (consider both teaching and learning)?*  *What have I learned from this lesson about the class or individuals that will inform my next lesson?* |
| **Next steps**  **What will I teach next based on learners’ understanding of this lesson?** |

# Changes to this Scheme of Work

This Scheme of Work has been amended. The latest Scheme of Work is version 2.0, published January 2021.

* The definition of the Thinking and Working Mathematically characteristic **TWM.03 Conjecturing** has been changed to: Forming mathematical questions or ideas.
* The definition of the Thinking and Working Mathematically characteristic **TWM 04 Convincing** has been changed to: Presenting evidence to *justify* or *challenge* a mathematical idea or solution.

There may be other minor changes that do not affect teaching and learning.

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